On the upscaling of a class of nonlinear parabolic equations

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Abstract
In this talk we develop an upscaling method for the following nonlinear parabolic equation

$$\partial_t b(u_\varepsilon) - \nabla \cdot (g^\varepsilon(x, u_\varepsilon) + a^\varepsilon(x, u_\varepsilon) \nabla u_\varepsilon) = f(x, t),$$

which stems from various applications of flow transport in porous media. Our direct motivation is the Richards equation which models the flow transport in the unsaturated porous media. We provide a detailed convergence analysis of the method under the assumption that the oscillating coefficients are periodic. While such a simplifying assumption is not required by our method, it allows us to use homogenization theory to obtain the asymptotic structure of the solutions. Numerical experiments are included to demonstrate the efficiency and accuracy of the proposed method. This is a joint work with Weibing Deng and Huang Ye.