Non Linear Analysis of Spatial Time Series

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Introduction

• Agenda

• Motivation
  – Global climate change
  – Environmental science
  – Ecological science

• Contents
Contents

- Description of STARMA models
- Modeling procedure
- Order determination
- Application to real data
- Summary and Extensions
STARMA Model

- Pfeifer and Deutsch (1980)
- R. v. Z measured at N sites
- Incorporates spatial dependencies
  - Matrix of weights
    - System of neighborhoods
\[ \text{STARMA}\left(\lambda_1, \lambda_2, \ldots, \lambda_p, q, m_1, m_2, \ldots, m_q\right) \]

\[ z(t) = \sum_{k=1}^{p} \sum_{l=0}^{\lambda_k} \phi_{kl} W^{(l)} z(t-k) - \sum_{k=1}^{q} \sum_{l=0}^{\lambda_k} \theta_{kl} W^{(l)} \varepsilon(t-k) + \varepsilon(t) \]

- \(Z(t)): \mathbb{N} \times 1, \ t=1, \ldots, T\)
- \(W^{(1)}): \mathbb{N} \times \mathbb{N}\)
- \(\varepsilon(t) \sim N(0, G)\)
- \(G = \sigma^2 I\)
Weighting Matrices $W^{(1)}$

- Fixed beforehand
- Reflect physical features
- Regular spaced systems:
  - equal weights
- Irregularly spaced systems
  - inversely proportional to distance
Modelling procedure

- Identification
- Estimation
- Diagnostic Checking
Identification

• Space-time autocovariance function

$$\gamma_{lk}(s) = E\left\{ \frac{[W^{(l)} z(t)] [W^{(k)} z(t + s)]}{N} \right\}$$

• Space-time autocorrelation function

$$\rho_l(s) = \frac{\gamma_{l0}(s)}{[\gamma_{ll}(0) \gamma_{00}(0)]^{1/2}}$$

• Sample estimates

$$\hat{\gamma}_{lk}(s) = \frac{1}{N} \sum_{i=1}^{T-s} \frac{[W^{(l)} z(t)] [W^{(k)} z(t + s)]}{T - s}$$
Identification

- Space-time analogue of Yule-Walker equations

\[ \gamma_{s0}(s) = \sum_{j=1}^{k} \sum_{l=0}^{\lambda} \phi_{jl} \gamma_{hl}(s-j) \]

\[ s = 1, \ldots, k \]

\[ h = 0, \ldots, \lambda \]

Space-time partial autocorrelation function: \( \phi_{k\lambda} \)
Estimation

• Assumption: orders are known
• Conditional maximum likelihood estimation
• $\Phi=(\phi)_{kl}$, $\Theta=(\theta)_{kl}$, $\sigma^2$ maximize:

$$l_*(\Phi, \Theta, \sigma^2) = -\frac{TN}{2} \ln(2\pi) - \frac{TN}{2} \ln(\sigma^2) - \frac{S_*(\Phi, \Theta)}{2\sigma^2}$$

Minimize $$S_*(\Phi, \Theta) = \varepsilon' I \varepsilon = \sum_{i=1}^{N} \sum_{t=1}^{T} \varepsilon_i^2(t)$$
Estimation Issues

• Nonlinear nature of the problem with STMA terms
• Initial estimates
• Order determination
Initial Estimates and Order Determination

- Hannan and Rissanen (1982) - ARMA
- Extension to STARMA
  - High order STAR(k₁,...,1)
- Y-W
  \[ BIC = \ln(\hat{\Sigma}) + 2m \frac{\ln(T)}{T} \]
  - Determine residuals
  - Write model in general linear form: \( Y = X\beta + \epsilon \)
    - \( \Phi,\Theta:(p,q) \) minimize BIC
    - Numerical estimation of \( \Phi,\Theta \)
Diagnostic Checking

- 95% confidence intervals for the residuals space-time autocorrelations

\[ \text{var}(\hat{\rho}_{t_0}(s)) \approx \frac{1}{N(T-s)} \]

- Test the hypotheses that \( G = \sigma^2 I \)

  - Use \( M = T^{-1} \sum \hat{\epsilon}_t \hat{\epsilon}_t' \)
Test the Hypotheses G=D

- Sample covariance matrix of the residuals
  \[ M = T^{-1} \sum \hat{\varepsilon}_i \hat{\varepsilon}_i' \]

- Test statistics:
  - H0: G=\sigma^2I
  - H0: G=D

\[ U = \frac{\left| M \right|}{\left[ \frac{tr(M)}{N} \right]^N} \quad U = \frac{\left| M \right|}{\prod_{i=1}^{N} M_{ii}} \]
Data

• Monthly mean temperatures 1951-1966
Data
Identification

- Assumption: $G = \sigma^2 I$

Space-time autocorrelation function of original data
Identification

Space-time autocorrelation function of deseasonalised series
Identification

Space-time partial autocorrelation function of
deseasonalised series
Estimation Results

• Assumption : $G = \sigma^2 I$
• STAR(24) spatial order 1
Diagnostic Checking

Space-time autocorrelation function of the residuals
Diagnostic Checking

Space-time partial autocorrelation function of the residuals

Hypotheses $G = \sigma^2 I$ rejected
Identification

- Assumption: general G

Transformed Space-time autocorrelation function of the deseasonalised data
Diagnostic Checking

Space-time autocorrelation function of the residuals
Summary and Extensions

- Stationarity
- Gaussianity
- Linearity

- Periodically correlated spatial time series
- Non-gaussian – Pareto distribution
  - Subba Rao and Wong (1997)
- Non-linear – Bilinear
  - Dai and Billard (1998)