Detonation Front Models: Theories and Methods

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Detonation Propagation

- combustion supported shock wave
- shock provides energy transport and reaction ignition
- inertial confinement (shock) yields high pressures and fast chemical reactions

\[ \eta \]

\[ D_n \]

\[ n_{rz} \]

\[ L \]

\[ X \]

\[ D_{cj} \]

- CJ is zeroth order model
Multi-Scale Issues - 1

- consider the detonation of a cylinder of high explosive

- disparate length scales \((L_{\text{eng}} \gg L \gg \eta_{rz})\)
- reaction rate effects important even when \(L \gg \eta_{rz}\)
- finiteness of \(\eta_{rz}/L\) influences propagation speed, \(D_0\), and pressure of detonation through edge effects
• $D_0$ decreases as $\eta_{rz}/L$ increases
• $D_0$ decreases as confinement decreases
• detonation extinction and a host of other phenomena depend on the value of $\eta_{rz}/L$
• CJ model doesn’t describe these effects
DNS of Detonation

• given a well calibrated “reactive Euler model,” these effects can be calculated via direct numerical simulation (DNS)

• resolved DNS requires fine zoning

• $O(50 \text{ pts})$ are needed in the reaction zone to get $D_0$ converged to $O(0.02 \text{ mm/us})$ using modern, hi-res methods. Multistep reaction models require more resolution. Maximum error of 1% in computed $\eta_{rz}$
DNS Cost Estimates

- instantaneous volume of 3D reaction zone, $V_{rz}$, is $V_{rz} \sim (L_{eng})^2 \eta_{rz}$. For $L_{eng} \sim 300\text{mm}$, $\eta_{rz} \sim 1\text{mm}$ and 50 points in the $\eta_{rz}$ direction, $O(10^{10})$ cells in the reaction zone at any instant
- with $\Delta t = 4 \times 10^{-3} \mu s$ and a problem time of $50 \mu s$ gives $1.25 \times 10^4$ time steps
- grind time of $10^{-4} \text{ s/cell/cycle}$
- computation time for the reaction zone only
  \[ T_{cpu3D} \sim (10^{-4})(10^{10})(1.25 \times 10^4) = 1.45 \times 10^5 \text{ days} \]
- 1,000 cpu perfect parallelization, $T_{cpu3D||} = 145 \text{ days}$
- with AMR, 2D calculations may be feasible
- a subscale model of detonation propagation is indicated
Detonation Shock Dynamics (DSD)

- detonation is supersonic combustion, **insulated** from the combustion products flow
- principally, detonation communicates with its surroundings laterally (sideways flow)
- the fundamental balance that sets the normal speed of detonation, $D_n$, is the interaction between chemical heat addition and flow divergence in the reaction zone
DSD - 2

- a subscale model for the motion of the detonation front, decoupled from the post reaction zone flow
- the fundamental balance is described by the product, $(\eta_{rz} \kappa)$, where $\kappa$ is the shock curvature. Typically, $(\eta_{rz} \kappa) = O(\varepsilon)$, with $\varepsilon \ll 1$, which defines a perturbation parameter for an asymptotic analysis

\[
\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0,
\]
\[
\frac{\partial \rho \vec{u}}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u} \vec{u} + \vec{I} P) = 0,
\]
\[
\frac{\partial \rho e}{\partial t} + \vec{\nabla} \cdot [(\rho e + P) \vec{u}] = 0,
\]

\[
E(P, \rho, \lambda) = \frac{P/\rho}{\gamma - 1} - q \lambda, \quad \frac{d\lambda}{dt} \equiv R = k (1 - \lambda)^\mu \left( \frac{P}{P_{cj}} \right)^n,
\]
• intrinsic (Bertrand) coordinate analysis of the reactive Euler equations rewritten in quasi-conservation form

\[
\begin{align*}
[\rho(D_n - u_\eta)]_{\lambda} &= -\frac{\mathcal{A}}{(R - \mathcal{L}(\lambda))}, \\
[\rho(D_n - u_\eta)^2 + P]_{\lambda} &= \frac{\mathcal{B}}{(R - \mathcal{L}(\lambda))}, \\
E + 1/2(D_n - u_\eta)^2 + \frac{P}{\rho} &= \frac{\mathcal{C}}{(R - \mathcal{L}(\lambda))}, \\
[u_\xi]_{\lambda} &= \frac{\mathcal{E}}{(R - \mathcal{L}(\lambda))},
\end{align*}
\]

\[
\begin{align*}
\mathcal{A} &= (D_n - u_\eta) \cdot (\mathcal{G} + \mathcal{L}(\rho)), \\
\mathcal{B} &= (D_n - u_\eta) \cdot ((u_\eta - D_n) \cdot \mathcal{G} + \rho \mathcal{H}) \\
&+ \mathcal{L}(\rho(u_\eta - D_n)) + \rho \cdot \mathcal{L}(D_n), \\
\mathcal{C} &= (D_n - u_\eta) \cdot (\mathcal{H} + \mathcal{L}(D_n)) + \frac{1}{\rho} \mathcal{L}(P) \\
&- \mathcal{L}(E + 1/2(D_n - u_\eta)^2 + \frac{P}{\rho}), \\
\mathcal{E} &= -\frac{P_\xi}{\rho(1 - \eta \kappa_s)} + \frac{u_\eta}{u_\xi} \mathcal{H} - \mathcal{L}(u_\xi), \\
\mathcal{G} &= \frac{\rho}{(1 - \eta \kappa_s)} (\kappa_s u_\eta + u_\xi \eta_\xi), \\
\mathcal{H} &= \frac{u_\xi}{(1 - \eta \kappa_s)} (D_n \xi - \kappa_s u_\xi), \\
\mathcal{L}^2 &= \epsilon \frac{D}{Dt} + \epsilon \frac{D\lambda^{(1)}}{Dt} \frac{\partial}{\partial \lambda} + O(\epsilon^{2+\delta}).
\end{align*}
\]
• $D_n(\kappa)$ scalings in the limit $(\eta_{rz} \kappa) = O(1)$
  \[ \tilde{t} = \epsilon^{\nu} t, \quad \tilde{\xi} = \epsilon^{\sigma} \xi, \quad \phi = \epsilon^{-\sigma} \tilde{\phi} \]

• with $\kappa = \tilde{\phi}, \tilde{\xi} = O(1)$

• when $\nu$ and $\sigma$ are sufficiently large, time and transverse spatial variations are neglected to get the leading order, quasi-steady nozzle theory

\[
\left[(D_n - u_\eta)^2 - c^2\right] \frac{du_\eta}{d\lambda} = \frac{E_{,\lambda} (D_n - u_\eta)}{\rho E_{,P}} - \frac{c^2 u_\eta (D_n - u_\eta) \kappa}{R}
\]

which with the Bernoulli equation, $R$, and the shock conditions defines a shooting problem for $D_n(\kappa)$

• $D_n - \kappa$ theory
- $u_\eta$ vs $\lambda$ phase plane

- ZND Profiles for Steady 1D Wave
  - Physical EOS

- Dn(K) for Constant Rate ($k = 1.1735 \text{ us}^{-1}$)
  - Physical EOS

- Physical EOS
  - $\rho_0 = 2 \text{ gm/cc}$, $\gamma = 3$, $q = 4 \text{ mm}^2/\mu\text{s}^2$
  - $R = 1.1735/\mu\text{s}$, $0 < \lambda < 1$
  - $R = 0$, $\lambda > 1$

- eos and rate model
- the front propagation law is $D_n(\kappa)$. Parabolic PDE for front dynamics

\[
\mathcal{L}(\phi) = -\frac{dD_n}{d\kappa} \frac{\partial^2 \phi}{\partial \xi^2}.
\]
**D_n(κ) Boundary Conditions - 1**

- nozzle ODEs apply only away from boundaries
- boundary layer exists near interfaces
  -- governed by “steady” elliptic (hyperbolic) PDEs for subsonic (supersonic) flows
- D_n(κ) front dynamics requires boundary condition on φ
- boundary layer solutions
  -- shock polar analysis
  -- solution of PDEs for small streamline deflection
\( D_n(\kappa) \) Boundary Conditions - 2

- shock polars

**Strong Confinment**

\[ \phi_e = 8.25, \quad \phi_{DSD} = 9.65 \]

\[ \omega = \pi/2 - \phi \]

**Weak Confinment**

\[ \phi_e = \phi_{DSD} = 35.3 \]
\( \text{D}_n(\kappa) \) Boundary Conditions - 3

- direct numerical simulation
  -- strong confinement

- boundary layer-type analysis gives relation for transforming \( \theta_e \) to \( \theta_{eDSD} \)

\[
\tan(\theta_{eDSD}) = L \tan(\theta_e)
\]

\( L = 1.155 \), constant rate HE
**D_n(K) Boundary Conditions - 4**

- **direct numerical simulation**
  -- nonclassical case, with inert “pulling” the explosive

![Graph showing pressure and shock angle variations with r(mm) and phi](image)

- **Nonclassical Confinement**
  - Supersonic Explosive Match

- **Shock Angle**
  - State-Independent Rate Explosive Low Density Confinement

![Graph showing pressure and shock angle variations with r(mm) and phi](image)
in summary, the DSD boundary algorithm for $\omega$ is:

-- compute $\omega_s$ for the sonic shock state

-- compute $\omega$ using shock polars for the interface inert

if $\omega$ is larger than $\omega_s$ call this $\omega_c$

modify $\omega_c$ following the boundary layer analysis

-- measure the $\omega$ of the front at the interface

when $\omega < \omega_s$ then apply extrapolation, $\vec{t}_s \cdot \vec{\nabla} \omega = 0$

when $\omega > \omega_s$ then set $\omega = \omega_c$ or $\omega = \omega_s$, which ever gives the lower match pressure
D_{n}(\kappa) Shock Shape Prediction

- comparison of a DSD and DNS calculated shock

![Shock Shapes for a R_e = 24mm Cylinder](image)

**Shock Shapes for a R_e = 24mm Cylinder**

- DNS vs DSD
- DNS - highest resolution
- DNS - lowest resolution
- DSD - phi=.168
- z (mm) - DSD, phi=.168
- z (mm) - DNS(dx=.0625)
- z (mm) - DNS(dx=.125)
- z (mm) - DNS(dx=.25)
- z (mm) - DNS(dx=.5)
DSD Front-Flow Coupling

**Design Principles**

- the dynamics of DSD fronts is decoupled from the following flow
- the following flow depends crucially on the front motion
  -- the proper detonation energy must be delivered to the flow
  -- the detonation exit state (pressure, density, velocities) seeded into the following flow must be correct
  -- mass, momentum and energy conservation must be maintained
- small inconsistencies between the following flow and the front motion must not be permitted to influence the front motion
  -- the DSD front must act as a trigger wave that is the first signal propagated into the unburnt explosive
- DSD reaction zone
DSD Reaction Zone - 1

- equation of state for the DSD reaction zone

\[ E(P/\lambda, \rho) = \frac{P\rho}{(\gamma - 1)\lambda}, \quad \text{with} \quad E_0 = \frac{D_{CJ}^2}{2(\gamma^2 - 1)} \]

\[ \frac{\rho}{P} \left( \frac{\partial P}{\partial \rho} \right)_s = 1 + (\gamma - 1)\lambda \]

\[ c^2 = (1 + (\gamma - 1)\lambda) \frac{P}{\rho} \]

- reaction rate for the DSD reaction zone

\[ R = \frac{D_t}{\eta_{prz}}, \quad 0 \leq \lambda \leq 1 \]

\[ R = 0, \quad \lambda \geq 1 \]
DSD Reaction Zone - 2

- analysis of the reaction-zone structure problem for the DSD reaction zone ($D_t$, $\kappa$ and $\eta_{prz}$ specified)

\[
\left[(D_t - u_\eta)^2 - c^2\right] \frac{du_\eta}{d\lambda} = \frac{(D_t - u_\eta)c^2}{(1 + (\gamma - 1)\lambda)\lambda} - \frac{c^2 u_\eta(D_t - u_\eta)\kappa}{R},
\]

- trigger supported reaction zone structures
  -- the ratio $(\kappa\eta_{prz})$ controls the profile
  (the location of the internal shock, $\lambda^*$)

![Graph showing the effect of $\kappa\eta_{prz}$ on the reaction zone profile]

$D_t = 7.2\text{mm/us}$
DSD Reaction Zone - 3

- exit state controlled principally by $D_t$ and secondarily by $\eta_{prz}$. The exit value of $u_\eta$ is exact

- exit state from the physical model and the DSD reaction zone can be made identical
Implementation: Front Algorithm

• fast-tube level-set front propagator
  -- complex topologies
  -- efficient

high order smoothness needed in level-set field for accurate solutions

**LS PDE**
\[
\frac{\partial \phi}{\partial t} + D_n(\kappa) = 0,
\]

**redistance PDE**
\[
\frac{\partial \phi}{\partial \tau} = S(\phi) \cdot \left[ 1 - |\vec{\nabla} \phi| \right], \quad S(\phi) = \frac{\phi}{\sqrt{\epsilon^2 + \phi^2}},
\]

fast-tube algorithm is $O(N)$ faster. Huygens problem scales like $O(N^2)$

<table>
<thead>
<tr>
<th>$N \times N$</th>
<th>time</th>
<th>$N^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>150x150</td>
<td>~10 s</td>
<td></td>
</tr>
<tr>
<td>300x300</td>
<td>~50 s</td>
<td>2.3</td>
</tr>
<tr>
<td>600x600</td>
<td>~335 s</td>
<td>2.7</td>
</tr>
<tr>
<td>1200x1200</td>
<td>~2350 s</td>
<td>2.8</td>
</tr>
</tbody>
</table>
Implementation: Reaction & Linking

- DSD uses a pseudo reaction zone -- $\lambda$ is prescribed function of time

$$\lambda = \min \left[ 1.0, \max \left( 0.0, \left( \frac{t - t_B}{t_E - t_B} \right) \frac{1}{b_{fc}} \right) \right],$$

$$b_{fc} = \max \left( 1.0, \frac{n_{prz}}{D_t \cdot (t_E - t_B)} \right)$$
Implementation: Examples

• features:
  -- multiple contacting or isolated explosive regions
  -- detonation across explosive/explosive boundaries
  -- interfaces with multiple inerts
  -- complex $D_n(\kappa)$ forms
  -- complex explosive shapes
  -- many, nonsimultaneous detonation ignition points
Model Explosive: DSD vs DNS - 1

- direct comparisons on the detonation cylinder for the model constant rate explosive copper confinement

- DSD uses the $D_n(\kappa)$ relation shown, $\omega_s = 0.9553$, $\omega_c = 1.4386$ and the physical reaction-zone length of $\eta_{rz} = 4$ mm ($D_0 - DSD = 6.805$ mm/$\mu$s -- exact)

<table>
<thead>
<tr>
<th>resolution</th>
<th>$D_0$ - DNS</th>
<th>$D_0$ - DSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>80x310 (1.0 mm)</td>
<td>6.769 mm/$\mu$s</td>
<td>6.729 mm/$\mu$s</td>
</tr>
<tr>
<td>160x620 (0.5 mm)</td>
<td>6.739 mm/$\mu$s</td>
<td>6.807 mm/$\mu$s</td>
</tr>
<tr>
<td>320x1240 (0.25 mm)</td>
<td>6.739 mm/$\mu$s</td>
<td>6.792 mm/$\mu$s</td>
</tr>
<tr>
<td>640x2480 (0.125 mm)</td>
<td>6.739 mm/$\mu$s</td>
<td>6.802 mm/$\mu$s</td>
</tr>
</tbody>
</table>
Model Explosive: DSD vs DNS - 2

- comparison of axial pressure and tangential acceleration of copper liner

![Diagram showing the comparison of axial pressure and tangential acceleration in a dynamic experiment.](image)

**Standard Cylinder Test (speed for 2 Cu particles)**

**Axial Pressure Comparison**

- DNS
- DSD

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Calibration to a Solid Explosive

- direct calibration of $D_n(\kappa)$ to experimental wave shape and velocity data gathered on explosive cylinders of various radii and colliding detonation

- assume a unique $D_n(\kappa)$ exists

\[
\frac{D_n}{D_{CJ}} = 1 - B \cdot \kappa \cdot \frac{(1 + C_2 \cdot \kappa^{e2} + C_3 \cdot \kappa^{e3})}{(1 + C_4 \cdot \kappa^{e4} + C_5 \cdot \kappa^{e5})}
\]

- Levenberg-Marquardt least-squares fit
Predictions using $D_n(k)$ Law

- calibration results,
  - $D_{CJ} = 7.764 \text{ mm/µs}$
  - $\omega_s = 0.8674$
- prediction on planar arc

<table>
<thead>
<tr>
<th>phase velocity</th>
<th>exp</th>
<th>DSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_0_{in}(\text{mm/µs})$</td>
<td>7.188</td>
<td>7.198</td>
</tr>
<tr>
<td>$D_0_{out}(\text{mm/µs})$</td>
<td>9.953</td>
<td>9.961</td>
</tr>
</tbody>
</table>
Predictions using DSD

- comparison with data on cylinder test for a solid explosive

DSD Calculation of Standard Cylinder Test

\[ D_0(\text{exp}) = 0.759 \text{ cm/us} \]
\[ D_0(\text{unconfined}) = 0.760 \text{ cm/us} \]
\[ D_0(\text{Cu}) = 0.762 \text{ cm/us} \]

data limit of experiment

\[ u_r(\text{cm/us}) \]
\[ r(\text{cm}) \]
Higher Order DSD Theory - 1

• simple arguments indicate time dependence may be important
  \[ \mathcal{L}(D_n) = -(D_0 \sin(\phi))^2 \kappa \]

• general scalings, with \( \kappa = O(\varepsilon) \)
  \[ \tilde{t} = \varepsilon^\nu, \quad \tilde{\xi} = \varepsilon^\sigma \xi, \quad \phi = \varepsilon^{1-\sigma} \tilde{\phi}, \]
  \[ D = \varepsilon^\beta \tilde{D} = (D_n - D_{CJ})/D_{CJ}, \]

• scaled result
  \[ \varepsilon^{\nu+\beta} D_{CJ} \tilde{\mathcal{L}}(\tilde{D}) = -\varepsilon D_0^2 \left( \sin(\varepsilon^{1-\sigma} \tilde{\phi}) \right)^2 \frac{\partial \tilde{\phi}}{\partial \tilde{\xi}}, \]

• DSD scalings, based on \((D_n/D_{CJ} - 1)\) and \( \kappa \) being \( O(\varepsilon) \)
  \[ \tilde{t} = \varepsilon t, \quad \tilde{\xi} = \varepsilon^{1/2} \xi, \quad \phi = \varepsilon^{1/2} \tilde{\phi}, \quad D = \varepsilon \tilde{D} = (D_n - D_{CJ})/D_{CJ} \]
  \[ \kappa = O(\varepsilon), \quad \mathcal{L}(D_n) = O(\varepsilon^2) \]
Higher Order DSD Theory - 2

• consider $\kappa$ as a **dependent** variable with the expansion
  
  $$\kappa(D) = \epsilon \kappa^{(1)} + \epsilon^2 \kappa^{(2)} + \cdots$$

• seek a formal expansion of $Y = (\rho, u_\eta, P)^T$ of the form
  
  $$Y = Y^{(0)} + \epsilon Y^{(1)} + \epsilon^2 Y^{(2)} + \cdots$$

  $$u_\xi = \epsilon^{3/2} u_\xi^{3/2} + \cdots$$

  $$\lambda = \lambda^{(0)} + \epsilon \lambda^{(1)} + \cdots$$

• where we can solve for $\lambda^{(1)}$ knowing only $R^{(0)}$
  
  $$\lambda^{(1)} = \tilde{D} \tilde{L}(\eta) = \tilde{D}L(\lambda^{(0)}) = \tilde{D}L(\lambda)$$

• to get an explicit expression for the time derivative in our transformed coordinates
  
  $$\tilde{\mathcal{L}}(\ ) = \epsilon \frac{D}{Dt} + \epsilon \frac{D \lambda^{(1)}}{Dt} \frac{\partial}{\partial \lambda} + O(\epsilon^{2+\delta})$$

• where
  
  $$\frac{D}{Dt} = \frac{\partial}{\partial t} + D_n \widehat{n} \cdot \vec{\nabla}$$
Higher Order DSD Theory - 3

- using the intrinsic coordinate equations, at $O(\varepsilon)$ we get
  \[
  \bar{M} \cdot \bar{Y}^{(1)} = \bar{N}^{(1)},
  \]
- a solvability condition involving $\bar{M}^{-1}$ returns the condition
  \[
  - \frac{D_{cj}\kappa^{(1)}}{k} = \left( \frac{\gamma + 1}{\sqrt{2\gamma}} \right)^2 \left( \frac{h}{2^h - 1} \right) \bar{D} = \frac{\bar{D}}{\alpha},
  \]
- at $O(\varepsilon^{3/2})$, we get
  \[
  u^{(3/2)}_{\xi, \lambda} = -\frac{1}{R^{(0)}\rho^{(0)}} P^{(1)}_{,\xi} + \frac{D_{cj}u^{(0)}_\eta}{R^{(0)}} \bar{D}, \bar{\xi},
  \]
- and finally at $O(\varepsilon^2)$, we get
  \[
  \bar{M} \cdot \bar{Y}^{(2)} = \bar{N}^{(2)},
  \]
- which returns via a solvability condition, $\kappa^{(2)}$
  \[
  \kappa^{(2)}(\bar{D}^2, \frac{D\bar{D}}{D\bar{t}}, \frac{\partial^2\bar{D}}{\partial\bar{\xi}^2}).
  \]
- combining and suppressing $\varepsilon$-dependence gives
  \[
  \bar{\kappa} = \mathcal{F}(\bar{D}) - A \frac{D\bar{D}}{D\bar{t}} + B \frac{\partial^2\bar{D}}{\partial\bar{\xi}^2}, \quad \bar{\kappa} = \frac{\kappa_s D_{cj}}{k}, \bar{t} \equiv kt, \bar{\xi} \equiv \frac{k}{D_{cj}}\xi,
  \]
Higher Order DSD Theory - 4

- replacing series by “likely” closed form expressions
  \[ 1 - 2\mathcal{D} + 3\mathcal{D}^2 + \cdots = (1 + \mathcal{D})^{-2}, \]
  \[ -\mathcal{D} - m\mathcal{D}^2 + \cdots = -\mathcal{D}(1 + \mathcal{D})^m, \]
- finally yields a DSD propagation law through \( O(\varepsilon^2) \)
  \[
  (1 + \mathcal{D})^{1-2n} \tilde{\kappa} = -\frac{\mathcal{D}(1 + \mathcal{D})^{1-2n-1}}{\alpha} + B \frac{\partial^2 \mathcal{D}}{\partial \xi^2} \\
  - \left( A + \frac{3(\gamma + 1)}{2\gamma} \left((1 + \mathcal{D})^{-1-2n} - 1\right) \right) \frac{D \mathcal{D}}{Dt}. \]
- like \( D_n(\kappa) \), this propagation law has parabolic dynamics
- both the shape (say as \( \phi \)) and \( D_n \) must be given initially
- a boundary condition on \( (\phi + bD,\xi) \) must be given
- we use this model to predict steady detonation in cylinders
- we specify \( \phi \) at the edge and \( \kappa \) on the axis
we examine the following two cases and compare results with high-resolution DNS

\[ \begin{align*}
\text{case-1} & : & n = 0 , & \mu = 0.5 , & k = 2.5147 \mu s^{-1} \\
\tilde{f}_2 = 2.6581 , & A = 1.332 , & B = 0.2024 \\
\text{case-2} & : & n = 2 , & \mu = 0.5 , & k = 1.2936 \mu s^{-1} \\
\tilde{f}_2 = -1.3636 , & A = 3.821 , & B = 0.2148
\end{align*} \]

we consider 3 limits, the full model (Dnxixi), Dndot and DnK, for their predictions
we stress the model by making \( \phi_e \) large (unconfined) and \( D_0 \) far from \( D_{CJ} \)
Higher Order DSD Theory - 6

- comparison of $D_n$ vs $\kappa$ along a shock for various models and conditions

- comparison of DSD with DNS for detonation phase velocity curves
Higher Order DSD Theory - 7

- higher order theory gives good prediction of very sensitive variables on difficult case
D_n(\kappa, ...) Law vs Euler Models

- real heterogeneous solid explosives are complex
- grain size can be of the order of the reaction length
- competition between second order effects determines 2D propagation
- empirical reaction rates are not “physically” correct at level of second order effects
- D_n(\kappa, ...) laws obtained directly from experiment with support from theory
Summary

• an overview of the elements of the DSD model
• elements of a DSD implementation
• examples of how DSD compares with DNS
• ability to predict results of physical experiments
• higher order theory
• argument on why subscale, calibrated models are needed to get highly accurate predictions of detonation propagation