Information goods (books, videos, journals) are sometimes bought and sometimes shared, rented, or loaned.

**Examples**

- English circulating libraries (1800s).
- Video stores (1980s).
- License servers (1990s).
- Interlibrary loan.
- Rights management systems (200x)
1. Model 1: simple case

\[ r(y) = \text{willingness to pay of individual } y \text{ to read/view book/video} \]

\[ cy + F = \text{cost of producing } y \text{ books} \]

*For sale only*

Profit maximization problem

\[ \max_y r(y)y - cy - F \]

has solution \( y^* \)
Sharing

Form club with \( k \) members. Transactions cost of sharing is \( t \). Number of books produced is \( x \), number read is \( kx \).

Assumption 1. Equal payments: so the wtp of club is \( k \) times wtp of member with lowest wtp. (Video rental.)


Assumption 2. Efficient club formation: the wtp of people in clubs that consume the good exceeds the wtp of people in clubs that don’t purchase the good.
Numerical example with individual purchase

6 consumers with wtp [9, 8, 7, 6, 5, 4], \( p = 6 \) implies 4 consumers buy.

Numerical example with group purchase

3 groups form: [(9, 8), (7, 6), (5, 4)], leading to wtp of [16, 12, 8]

\( p = 12 \) implies 2 groups (= 4 consumers) buy.

Another group formation: [(9, 6), (8, 7), (5, 4)], leading to wtp of [12, 14, 8]

\( p = 12 \) implies 2 groups (= 4 consumers) buy.
Demand curve

- Individual demand
- Group demand

Price vs. Price/3
Analysis

Implies willingness to pay of club to buy book is

\[ b(x) = kr(kx). \]

If transactions cost to sharing, then

\[ b(x) = k[r(kx) - t]. \]

Profit maximization problem

\[ \max_x b(x)x - cx - F \]
Becomes:
\[
\max_x r(kx)kx - \left( t + \frac{c}{k} \right) kx - F.
\]

Letting \( y = kx \), we have
\[
\max_y r(y)y - \left( t + \frac{c}{k} \right) y - F. \tag{1}
\]

Let \( y' \) be solution to this problem.

Then \( y' > y^* \) iff
\[
t + \frac{c}{k} < c,
\]

which can be written as
\[
t < c \left[ \frac{k - 1}{k} \right]. \tag{2}
\]
Special Cases

$k$ large: reduces to $t < c$.

In this case sharing results in more books being read, lower price per reading, higher profits to producer. Why? Sharing is a more efficient form of distribution.

$t = c = 0$: neutral. Same books read, same profit.

$t \gg c$: more books read without library than with it.
2. Generalization for purely digital goods

Baseline problem is

$$\max_y p(y)y.$$ 

Now let $y = \text{amount consumed}, \ x = \text{amount produced}, \ \text{and suppose value increases by more or less than } k \ \text{due to free rider problems, overestimation, etc. Specifically:}$

$$P(y) = ap(y)$$

$$y = bx.$$
Profit maximization for purely digital good:

\[ \max_x P(y)x. \]

Becomes

\[ \max_y ap(y)x, \]

or

\[ \max_y \frac{a}{b} p(y)y. \]

Note: amount of digital good viewed is independent of the sharing arrangement. Profits go up or down depending whether \( a \) is larger or smaller than \( b \). Intuition.
3. Model 2: different values

In previous model buying has same utility as renting. What if they are different? E.g., multiple views.

\[ u_b = \text{utility from buying video} \]
\[ u_r = \text{utility from renting video} \]
\[ b = \text{price to buy video} \]
\[ b/k = \text{price to rent video (once)} \]
\[ t = \text{transactions cost to renting} \]

Incentive compatibility constraint: if \( b \) is too large consumers will share
Producer prices to buy

Incentive compatibility

\[ u_b - b \geq 0 \]

\[ u_b - b \geq u_r - \frac{b}{k} - t \]

Rearrange:

\[ u_b \geq b \]

\[ \frac{k}{k - 1} [u_b - u_r + t] \geq b. \quad (3) \]

Interesting case is where second constraint binds.
Producer prices to rent

\[ u_r - \frac{b}{k} - t \geq 0 \]

\[ u_r - \frac{b}{k} - t \geq u_b - b \]

Rearranging these gives us

\[ k[u_r - t] \geq b \]

\[ b \geq \frac{k}{k-1}[u_b - u_r + t] \tag{4} \]

profit in buy market = \[ \frac{k}{k-1}[u_b - u_r + t] \]

profit in rental market = \[ u_r - t. \]
Note: in rental market profits are decreasing in \( t \); in buy market profits are increasing in \( t \).

**When is buy market more profitable?**

\[
\frac{k}{k-1} [u_b - u_r + t] > u_r - t, \\
\text{or} \\
\quad u_b > \left(2 - \frac{1}{k}\right) (u_r - t) \quad (5)
\]
Assume that $u_r = v$, $u_b = mv$. Then equation (5) becomes

\[
\left( m - 2 + \frac{1}{k} \right) v \geq - \left( 2 - \frac{1}{k} \right) t
\]

Conclusion: *if consumers will watch the 2 movie or more times, the buy market is more profitable.*
4. Endogenous group size

Suppose that \( t = w(k - 1) \). (Due to e.g., simple waiting model.)

Optimal club size:

\[
\min_k \ (k - 1)w + \frac{b}{k}
\]

Implies

\[
k^* = \sqrt{b/w},
\]

Obvious comparative statics.
Nash equilibrium in rental market

\[ k^2 = \frac{b}{w} \]

\[ b = k[u_r - w(k - 1)]. \]

Solution is

\[ k_{rent} = \frac{u_r + w}{2w} \]

\[ b_{rent} = \frac{(u_r + w)^2}{4w} \quad (6) \]
The profits to the producer are

\[
\text{profits in rental case} = \frac{u_r + w}{2}.
\]

Note: now profits are increasing in \( w \)!

Two effects of increase in \( w \) on profits: reduction in size of group and decrease in wtp.
Nash equilibrium in buy market

Want to raise price as high as possible without inducing (optimal) sharing. Solve:

\[ u_b - b = u_r - w(k - 1) - \frac{b}{k} \]

\[ k^2 = \frac{b}{w} \]

Economically appropriate solution:

\[ b = u_b - u_r + w + 2\sqrt{(u_b - u_r)w} \]

\[ k = 1 + \frac{\sqrt{(u_b - u_r)w}}{w} \]
When is buy market more profitable?

\[(u_b - u_r) + w + 2\sqrt{(u_b - u_r)w} > \frac{u_r + w}{2},\]

Reduces to:

\[2u_b - 3u_r + w + 4\sqrt{(u_b - u_r)w} > 0.\]

Surely holds when

\[u_b > \frac{3}{2}u_r.\]

If \(u_b = mv, u_r = v,\) then all we need is that

*if viewers watch the movie more than once, the buy market is more profitable.*
5. Heterogeneous tastes

Two groups, with values $v_H$ and $v_L$ for renting, $mv_H$ and $mv_L$ for owning, and transactions costs $t_H$ and $t_L$ with $t_H > t_L$. There are $H$ high-value people and $L$ low-value people and zero cost of production.

- sell only to high-value type: profit $= mv_H \times H$
- rent to both types: profit $= [v_L - t_L] \times (H + L)$
- sell to both types: profit $= mv_L \times (H + L)$ (more profitable than renting)
- sell to high-value, rent to low-value
Incentive compatibility:

\[ mv_H - b \geq 0 \]  \hspace{1cm} (7)

\[ mv_H - b \geq v_H - \frac{b}{k} - t_H \]  \hspace{1cm} (8)

\[ v_L - \frac{b}{k} - t_L \geq 0 \]  \hspace{1cm} (9)

\[ v_L - \frac{b}{k} - t_L \geq mv_L - b \]  \hspace{1cm} (10)
Combining (8) and (10)

\[
\left( \frac{k}{k-1} \right) \left[ (m - 1) v_H + t_H \right] \\
\geq b \geq \\
\left( \frac{k}{k-1} \right) \left[ (m - 1) v_L + t_L \right]
\]

Can always be satisfied.
Combining (11) and (7),

\[ b = \min \left\{ mv_H, \left( \frac{k}{k - 1} \right) [(m - 1)v_H + t_H] \right\} \]

Make approximation that \( k \) is large; equation becomes

\[ b \approx mv_H + \min\{0, t_H - v_H\} \]
Case 1. \( t_H > v_H \)

\( b^* \approx m v_H \): high-value buys at WTP

\( v_L - m v_H / k - t_L \geq 0 \): low-value rents

Case 2. \( t_H < v_H \)

\( b^* \approx (m - 1) v_H + t_H \)

Profits to selling and renting exceed profits from selling only when \( L/k \) large:

\[
[t_H - v_H] H + [(m - 1) v_H + t_H] \frac{L}{k} > 0.
\]
6. Implications

Markets for sharing can increase profits when

• transactions cost of sharing is less than cost of production;

• when users want to use product only once;

• when can be used for quality differentiation to segment market;