Geophysical Inversion for Mineral Exploration

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http://www.eos.ubc.ca/ubcgif
The **San Nicolas** massive sulfide deposit:

**north-facing geologic cross-section**

- **Mafic Volcanics**
- **Quartz Rhyolite**
- **Sulphide**
- **“Keel”**
- **Tertiary Breccia**

Elevation [m] from 1600 to 2000

Easting [m] from -2000 to -1100
Relevant Equations and Surveys

- Gravity: \[ \nabla^2 \phi = 4\pi G \rho \]
- Magnetics: \[ \nabla^2 \phi = \nabla \cdot M \]
- DC resistivity: \[ \nabla \cdot \sigma \nabla \phi = -I \delta(r - r_s) \]
- Time & frequency domain electromagnetics:
  \[ \nabla \times E = -\frac{\partial B}{\partial t} \quad \nabla \times H = J + \frac{\partial D}{\partial t} \]
Some Practical Considerations

- Poor resolution (potential & diffusive equations).
- Limited data.
- Inversion results wanted “quickly”.
- Results will be used in combination with other geological and/or geophysical information.
- Inversion results will be interpreted by non-experts.
Questions:

- How do we get the “best” inversion result?
- How do we characterize and convey information about non-uniqueness?
- How can we prevent over-interpretation of the inversion result?
- How do we carry out practical hypothesis testing?

To answer these questions, we pursue model construction.
Outline:

- Inversion Methodology.
- DC resistivity, and a field application.
- Limited exploration of model space.
- DOI (Depth of Investigation).
- Hypothesis testing.
- Conclusion.
The inverse problem

- Geophysical data are: \( F[m] + \varepsilon = d \)
  - \( m \): model --- unknown
  - \( F \): forward mapping operator
  - \( \varepsilon \): errors
  - \( d \): observations (data)

- Given:
  - data, errors, a forward modelling method

- Find:
  - plausible model(s) which generated measurements.
Inversion as an optimization problem

A principal difficulty in solving the inverse problem is the non-uniqueness

- Define
  - $\phi_m$: Model objective function.
  - $\phi_d$: Misfit function.

- Minimize

$$\phi = \phi_d + \beta \phi_m \quad \text{Subject to} \quad \phi_d < \text{Tol.}$$

$$0 < \beta < \infty \quad \text{is a constant}$$
Misfit functional

\[ \phi_d = \sum_{i=1}^{N} \left( \frac{F_i[m] - d_i^{obs}}{\varepsilon_i} \right)^2 \]

• What \( \phi_m \) to use?

• Desirable features include:
  – “Character” should emulate geology.
  – “Simple” … no unnecessary complexity.
  – Compatible with prior information.
  – Flexible.
A generic model objective function.

\[ \phi_m(m) = \alpha_s \int_s w_s (m - m_0)^2 \, dv + \alpha_x \int_s w_x \left( \frac{\partial(m - m_0)}{\partial x} \right)^2 \, dv + \ldots \]

- \( \alpha_s, \alpha_x \ldots \) are constants.
- \( w_s, w_x, \ldots \) are functions.
- \( m_0 \) is a reference model.
Numerical solution

- Discretize: Divide the earth into $M$ cells of constant physical property ($M \gg N$).

Minimize \[ \phi = \phi_d + \beta \phi_m \]

\[ = \left\| W_d (F[m] - d^{obs}) \right\|^2 + \beta \left\| W_m (m - m_0) \right\|^2 \]

- Use the Gauss-Newton method for solution.

- Solving for $\beta$:
  - Discrepancy principle.
  - GCV.
  - L-Curve.
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The DC resistivity experiment:

- Resistivity and IP survey configuration.
- Current converges on conductors, diverges from resistors.
- Charges accumulate.
- Charge distribution affects recorded potentials.

\[ \phi = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} \]
DC Resistivity: Forward modelling

• Governing equation:

\[ \nabla \cdot (\sigma \nabla \phi) = -I \delta(r - r_s) \]

• B.C.’s: \( \frac{\partial \phi}{\partial z} = 0 \) at surface

\( \phi \rightarrow 0 \) as \( |r - r_s| \rightarrow \infty \)

• Solve using finite volume method
Plotting the data

- “Pole-dipole” survey, \( n = 1\ldots8, \quad a = 10\text{m}, \)
- \( N = 316 \) data points.
- Datum is \( \rho_a = 2\pi n(n+1)a \frac{\Delta V}{I} \)
- “pseudosection” of synthetic data:
3D DC resistivity field example from MIM

Surface topography:

10 lines surveyed
Apparent conductivity data along 10 survey lines

Easting (m)

N-spacing

mS/m

1410.000
562.000
224.000
69.100
35.500
14.100
5.620
2.240
0.691
0.355
0.141
3D conductivity model recovered via 3D inversion:
Summary:

- Inversion methodology seems basically OK.
- We can do an adequate job if objects are large, and field data are good.

Non-uniqueness:

- What confidence do we have in the existence of the features?
- What level of detail can be inferred?
- Are there artifacts?

Our procedure:

- Alter the objective function and carry out further inversions.
Outline:

- Inversion Methodology.
- DC resistivity, and a field application.
- Limited exploration of model space: A 2D resistivity synthetic example
- DOI (Depth of Investigation).
- Hypothesis testing.
- Conclusion.
A synthetic inversion result

- \( N = 316, \ M = 2500 \)
- \((\alpha_s, \alpha_x, \alpha_z, m_0) = (0.001, 1, 1, 400 \ \Omega m)\)

Data with 5% Gaussian noise  
Predicted data

Recovered resistivity model
How severe is the non-uniqueness?

- Carry out a limited exploration of model space.

\[ \phi_m(m, m_0) = \alpha_s \int_s (m-m_0)^2 \, dx \, dz + \alpha_x \int_s \left( \frac{\partial m}{\partial x} \right)^2 \, dx \, dz + \alpha_z \int_s \left( \frac{\partial m}{\partial z} \right)^2 \, dx \, dz \]

- Parameters to vary are: \( \alpha_s, \alpha_x, \alpha_z, \) and \( m_0. \)

1. \((\alpha_s, \alpha_x, \alpha_z, m_0) = (0.001, 1, 1, 400)\)
2. \((\alpha_s, \alpha_x, \alpha_z, m_0) = (1, 0, 0, 400)\)
3. \((\alpha_s, \alpha_x, \alpha_z, m_0) = (0, 1, 0, 400)\)
4. \((\alpha_s, \alpha_x, \alpha_z, m_0) = (0, 0, 1, 400)\)
Recovered models using 4 versions of $\phi_m$

$$
\phi_m(m, m_0) = \alpha_s \int (m - m_0)^2 \, dx \, dz + \alpha_x \int \left( \frac{\partial m}{\partial x} \right)^2 \, dx \, dz + \alpha_z \int \left( \frac{\partial m}{\partial z} \right)^2 \, dx \, dz
$$
**Predicted data** from models using 4 versions of $\phi_m$

- (.001, 1, 1, 400)
- (1, 0, 0, 400)
- (0, 1, 0, 400)
- (0, 0, 1, 400)
Comments:

- Mathematical non-uniqueness can be quite severe.
- Some models are not geologically reasonable.

Find alternate models with “right” character by altering the reference model.
Other valid resistivity models:

“best” model: \( m_0 = 400 \ \Omega \text{m} \)

- \( m_0 = 4000 \ \Omega \text{m} \)
- \( m_0 = 40 \ \Omega \text{m} \)

\( m_0 = 400 \rightarrow 400,000 \ \Omega \text{m} \)

\( m_0 = 400 \rightarrow 0.4 \ \Omega \text{m} \)
Summary so far:

- A few inversions can provide considerable insight.

- Further information is available: e.g. DOI.
Outline:

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- Limited exploration of model space.
- DOI (Depth of Investigation).
- Hypothesis testing.
- Conclusion.
Quantifying depth of investigation

Model objective function

$$\phi_m = \alpha_s \int (m - m_{ref})^2 \, dxdy + \ldots$$

- For each pixel define

$$DOI = \frac{|m_1 - m_2|}{|m_{ref1} - m_{ref2}|} \quad 0 \leq DOI \leq 1$$

- Where DOI ≈ 0 result is controlled by the data.
- Where DOI ≈ 1 result is controlled by $\phi_m$
Depth of investigation index

- Generate a “D.O.I.” index by comparing resistivity models that used different $m_0$.

“best” model: $m_0 = 400 \, \Omega \cdot \text{m}$

$m_0 = 40 \, \Omega \cdot \text{m}$

**D.O.I. Index; 0 < value < 1**
Depth of investigation

- Use the D.O.I. to help establish where data no longer constrain the recovered model.
True and recovered models
Outline:

- Inversion Methodology.
- DC resistivity, and a field application.
- Limited exploration of model space.
- DOI (Depth of Investigation).
- Hypothesis testing using model construction.
- Conclusion.
How to test if features are required?

- Focus on a particular feature.
- Try to construct a counter example. That is, a model that has “acceptable” character, but does not have the feature.
  - **Success** implies reduced confidence in the feature.
  - **No success** implies increased confidence that the feature exists. However this is not conclusive proof.
How to test if features are required?

- Focus on a particular feature.

“Preferred” model recovered by inversion.

- Build a suitable weighting function $W$.

Weighting function $W$

$$
\phi_m(m) = \alpha_s \int_S w_s (m - m_0)^2 \, dv + \alpha_x \int_S w_x \left( \frac{\partial (m - m_0)}{\partial x} \right)^2 \, dv + \ldots
$$
Results of penalizing structure via weighting:

Weighting $W$

Result, $W = 100$

Result, $W = 10$

No weighting.
Outline:

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- DOI (Depth of Investigation).
- Hypothesis testing.
- Conclusion.
Conclusions: exploring model space

- Limited, but practical, insight can be obtained through a focussed exploration of model space.

- Constructed models can provide first order information about depth of investigation.
Conclusions: hypothesis testing

• Hypotheses about the model might be addressed by carrying out inversions with specifically tailored objective functions.

• In practice:
  – We need more information about the model to make decisions about whether our solution is potentially realistic.
  – For mineral problems this means combining geology, rock properties and geophysics information.
Outline:

- Inversion Methodology.
- DC resistivity, and a field application.
- Limited exploration of model space.
- DOI (Depth of Investigation).
- Hypothesis testing.
- Conclusion.
- Addendum
Addendum

- Eventhough we have incomplete and inaccurate data, and poor resolution, we can still obtain meaningful results if we can find the “right” objective function
- Example: 3D magnetic inversion
Magnetic Data

Total field anomaly:

\[ d = \vec{B}_a \cdot \hat{B}_0 \]

- \( \hat{B}_0 \) = direction of Earth’s field.
- \( \vec{B}_a \) = anomalous field of the block.

\[
d(r_i) = \int_V \kappa(\vec{r}) \left\{ \frac{B_0}{4\pi} \hat{B}_0 \cdot \nabla \nabla \left( \frac{1}{|\vec{r}_i - \vec{r}|} \right) \cdot \hat{B}_0 \right\} dv \\
= \int_V \kappa(\vec{r}) g_i(\vec{r}) dv
\]
A Synthetic Example

Synthetic Model:

- Block top = 50 m
- 100 x 100 x 100 m
- Susceptibility = 0.05
- I = 30°
- D = 45°
Synthetic survey:

Measured data:
- 100-m line spacing.
- 25-m station spacing.
- \( N = 175 \) (elevation= 2m).

Data as recorded.

Contoured “perfect” data

Contoured data with noise = 2 nT
Inversion Results

$N=175$, $M=11,492$ (25-m cubes)

Inversion proceeded normally ...

$\phi_d = 178.3$, $\phi^*_d = 175$
What went wrong?

- **Gauss’ theorem:** The observed magnetic field can be reproduced by an infinite number of susceptibility distributions, such as a thin layer of susceptibility at an arbitrary depth.
• **Nature of the kernels:** the kernel function decays rapidly with the depth. It is the easiest for the inversion to reproduce the data with susceptibility concentrated near the surface.

linear

log

poor model
Depth Weighting: 3D Magnetics

- Decays with depth:
  \[ w(z) = \frac{1}{(z + z_0)^{3/2}} \]

- \( z_0 \): by least-squares fit between \( g(z) \) and \( w^2(z) \)
Inversion using depth weighting

- The model objective function incorporating depth weighting is as follows:

\[
\phi_m^w(m) = \alpha_s \int [w(z) \kappa(\vec{r})]^2 \, dv + \alpha_x \int \left[ \frac{dw(z) \kappa(\vec{r})}{\partial x} \right]^2 \, dv + \cdots
\]

\[
\equiv \vec{\kappa}^T \mathbf{Z}^T \mathbf{W}^T \mathbf{W} \mathbf{Z} \vec{\kappa}
\]

- \( \mathbf{Z} \) is a diagonal matrix containing the discretized depth weighting.
Result of inverting with depth weighting

True model

Inverting with no depth weighting
Positivity

- Susceptibility is positive. This constraint should be included in the inversion.

- Many geophysical problems require positive physical properties.

- The inverse problem is stated as

\[
\min \phi = \phi_d + \beta \phi_m^w \\
\text{s.t. } \tilde{\kappa} \geq 0
\]

- The problem becomes non-linear.
Inversion proceeded normally …

\[ \phi_d = 177.1, \quad \phi^* d = 175 \]
Geological question: are outcrops connected at depth?