Time reversal imaging and communications in random media

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From Scientific American, November 1999 (M. Fink).
Range: $L$, Carrier wavelength $\lambda$, Array size $a = (N + 1)\lambda/2$. Source at $y$, Search point at $y_s$, Transducers at $x_p$. **Remote sensing regime:** $\lambda << a << L$. 
Cross-range (Rayleigh) resolution: $\frac{\lambda L}{a}$
Range resolution: $\frac{\lambda L^2}{a^2}$
Same as for optical instruments.

Wireless: $\lambda = 20cm$, $a = 2m$, $L = 5km$
Cross-range resolution: 500m

● What happens in a randomly inhomogeneous medium? SUPER-RESOLUTION
Numerical simulations

- Solve the wave equation (finite element, time domain) with a randomly fluctuating propagation speed about a constant mean.

- **Ultrasound probing:** $\lambda = 0.5\text{mm}$, $a = 2.5\text{mm}$ (5mm), $c_0 = 1.5\text{km/s}$.
Top deterministic. Bottom random. $s$ = standard deviation of the fluctuations of sound speed.
Multipathing causes super-resolution

- A source at $y$ radiates onto the array. The recorded signal is time reversed (conjugated in Fourier domain) and sent into medium. The back propagated field at the observation point $y^o$ is

$$\Gamma(y, y^o, t) = \int e^{-\imath \omega t} \sum_{j=1}^{N} \hat{G}(y, x_j, \omega) \bar{f}(\omega) \hat{G}(y^o, x_j, \omega) d\omega$$

- **Super-resolution**: In a random medium, because of multipathing the effective aperture $a_e \gg a$ or, $\frac{\lambda L}{a_e} \ll \frac{\lambda L}{a}$. 
Self averaging of back propagated field

- In a suitable asymptotic regime, decorrelation of \( \hat{\Gamma}(y, y^o, \omega) = \hat{f}(\omega)\hat{G}(y, x_j, \omega)\hat{G}(y^o, x_j, \omega) \) for frequencies \( \omega_1 \neq \omega_2 \) gives

\[
E \{ \Gamma^2(y, y^o, t) \} \approx E \{ \Gamma(y, y^o, t) \}^2.
\]

- Back propagated field is self averaging (ergodic):

\[
\Gamma(y, y^o, t) \approx E \{ \Gamma(y, y^o, t) \}.
\]

- The analytic expression for \( \Gamma(y, y^o, t) \) is close to the expression for the back propagated field in a homogeneous medium for an array of effective aperture \( a_e \sim \sqrt{a^2 + \gamma L^3} \gg a \).

- Self-averaging and super-resolution are observed in time only. Broadband is essential (BPZ-JASA-02). Distributed sources OK for narrow-band.
Applications, history of time-reversal

- **Optical phase conjugation, ’70s**: Adaptive optics, laser fusion, holography. Single frequency, distributed sources. Super-resolution not (easily) observed.

- **Underwater acoustics, late ’80s**: Super-resolution observed, communications, imaging (Kuperman, Jackson, Dowling).

- **Ultrasound, early ’90s**: Non-destructive testing, medical applications. Super-resolution observed very clearly (Fink).

- **Wireless**: Use multipathing to increase communication capacity. Radar imaging in cluttered environments (not possible with synthetic aperture radar).
Time reversal experiment with 20-transducer *Time Reversal Mirror* in the Mediterranean Sea (off Italy, at Formiche di Grosseto). 120m depth, 15-30km propagation distance.

Performed by the **Scripps Institution of Oceanography** (La Jolla, CA) and **SACLANT Undersea Research Center** (La Spezia, Italy). Published in JASA 1996, 1997 by Kuperman et.al.

From Scientific American, November 1999 (M. Fink).
Time reversal experiment with 20-transducer *Time Reversal Mirror* in the Mediterranean Sea (off Italy, at Formiche di Grosseto). 120m depth, 15-30km propagation distance.

Note: $L\lambda/a \sim 15000 \times 1/120 \sim 100m$. [No resolution? TR in a waveguide]

From from Scientific American, November 1999 (M. Fink).
Active array imaging.

An array of $N$ transducers probes a medium with $M$ small scatterers with a pulse $f(t)$ emitted from transducer $j$, located at $x_j$, and the echos $P_{jk}(t)$ are recorded at $x_k$, for $k = 1, \ldots N$.

**Imaging:** From the $N \times N$ response matrix $(P_{jk}(t))$ find the number $M \leq N$ of scatterers and their location $y_1, y_2 \ldots y_M$.

**Random media:** In the regime $\lambda \leq l \ll a = N\lambda/2 \ll L$, where there is significant **multipath**ing, how is imaging affected?
How to do imaging in random media

• We use the statistical stability and the super-resolution of time reversal in random media, for imaging.

• Identify the unknown targets as minima or maxima of objective functions that are self-averaging.

Motivate echo mode passive target array imaging with active target array imaging (locate an active source).

To find the location $y^*$ of that source we use the matched field method in the time domain, which works well in random media.

This means: time reverse and then back propagate in a homogeneous medium, numerically.
For a single active source

\[ y^* = \arg\min_z F(z, 0) \]

where

\[ F(z, t) = \mathcal{F}^{-1}\left\{ \left| (\hat{f}(\omega)\hat{g}(y, \omega) )^H \frac{\hat{g}_0(z, \omega)}{|\hat{g}_0(z, \omega)|} \right|^2 \right\} \]

where

\[ \hat{g}(y, \omega) = \left( \begin{array}{c} \hat{G}(y, x_1; \omega) \\ \vdots \\ \hat{G}(y, x_N; \omega) \end{array} \right) \]

where \( \hat{G}(y, x; \omega) \) is the (random) Green’s function. The vector \( \hat{g}(y, \omega) \) is the radiation vector.

The subscript zero indicates free space (deterministic medium).

The matched field functional is statistically stable.
Direction of arrival estimation

On the left time domain estimation, on the right fixed frequency. Use MUSIC (Multiple Signal Classification), a variant of Matched Field that performs a little better.
Imaging of one target in a random medium using MF and Arrival Time estimation (AT)

\[ s = 2.25\%,\text{ M.F.} = 3.9\% \]

\[ s = 2.53\%,\text{ M.F.} = 4.38\% \]

\[ s = 4.67\%,\text{ M.F.} = 8.10\% \]

\[ s = 3.82\%,\text{ M.F.} = 6.62\% \]

\[ s = \text{ standard deviation, M.F.} = \text{ max. fluctuations of sound speed} \]
Imaging of two targets in a random medium using MF and Arrival Time estimation AT

$s = \text{standard deviation, M.F. = max. fluctuations of sound speed}$
Communications with time reversal

Point (Transmit) to Point (Receive) communication:

1. Receiver sends probing pulse $f(t)$ of width $\delta$

2. Transmitter receives $h(t) = f \ast G(t)$

3. Transmitter time-reverses a piece $k_T(t) = h(T - t)$ for $0 \leq t \leq T$ and zero otherwise

4. Transmitter sends a message $s(t) = \sum_{j=1}^{M} n_j k_T(t - j \tilde{\delta})$, where the information is $\{n_j = 0, 1\}$ and the spacing $\tilde{\delta}$ is chosen suitably. Note the overlap

5. The receiver gets, essentially, $r(t) = \sum_{j=1}^{M} n_j f(t - j \tilde{\delta})$, multiplied by a spatially limiting function, as in super-resolution
Summary and Conclusions

- Time reversal with multipathing lends itself to many exciting, surprising and important applications.

- The mathematical infrastructure for understanding and using time reversal is largely underdeveloped. It involves the ergodic theory (asymptotic theory) of stochastic PDEs, signal processing, communication theory, diffraction theory, computational wave propagation, ...