Partitioned Kalman Filter & Smoother

A computationally efficient method of assimilation.

GIST: Approximate state error covariance matrix \( P \) by a sum of smaller independent elements that are evaluated separately.  \( (MWR, 2002, 1370-1383) \)

e.g., Partition state (dimension \( N \)) into \( L \) equal elements. Then each partition would require \( (N/L)^3 \) operations, with a combined overall operation count of \( N^3/L^2 \).

\[
\delta x \approx B_1 \delta x'_1 + \cdots + B_L \delta x'_L \approx \sum_i B_i \delta x'_i \\
\text{dim}(\delta x'_i) << \text{dim}(\delta x) \\quad i
\]

\( B_L \): State Transformation

\[
P \approx \sum_i B_i P'_i B_i^T, \quad P'_i \equiv \left\langle \delta x'_i \delta x'^T_i \right\rangle
\]
\[
P = \sum_{i}^{L} B_i P'_i B_i^T, \quad P'_i = \begin{pmatrix} \delta x'_i & \delta x'_i^T \end{pmatrix}
\]

\[
K = \sum_{i}^{L} B_i K'_i, \quad K'_i = P'_i H'_i^T R^{-1}
\]

\[
S = \sum_{i}^{L} B_i S'_i B_i^*, \quad S'_i = P'_i A'_i^T P'_i(-)^{-1}
\]

\[
\delta'_K \equiv K (y - Hx(-)) \approx \sum_{i} B_i \delta'_K, \quad \delta'_K \equiv K'_i (y - Hx(-))
\]

\[
\delta'_S \equiv x(+) - x \approx \sum_{i} B_i \delta'_S, \quad \delta'_S(t) \equiv S'_i \left(\delta'_S(t+1) + \delta'_K(t+1)\right)
\]
PKF/PS for an Ocean Model

• A global, high resolution (0.3°, 10m), non-linear ocean general circulation model.
• Partitioned global ocean into 7 regional cells.
• Model error source modeled as errors in wind.