Haptic Rendering of NURBS Models

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Sample Design Pipeline

Clay model is modified
Motivation - Virtual Prototyping

• Replace physical prototypes with digital ones.
• Allow human interaction with virtual models.
• Test
  – aesthetics
  – assemblility
  – ergonomics
Approach

• Add sense of touch to virtual prototyping system.
• Avoid conflicting cues between visual and haptic senses.
• Leverage innate human talents at manipulation.
Computing Haptic Forces

- Approximate complicated surface deformation with a *wall model*
- Force proportional to penetration depth
Local vs Global Closest Point

Why do we want to track the local closest pt?

- Local closest point
- Previous position
- Desired penetration depth

- Global closest point
Background - Surface Rep

• Choice of surface representation
  – Intermediate
  – Polygonal
  – Implicit
  – Voxel
  – NURBS
NURBS Representation

- Same as used in CAD
  - avoid translation
- Compact storage
- Exact (assembly)
- Thompson ‘97, Johnson ‘98, ‘99, Stewart and Buttulo ‘97
NURBS Surfaces

Control mesh

Control point
NURBS Background

• Piecewise-polynomials of two variables
• Control mesh weights the basis functions
• Refinement adds DOFs to surface
Trimmed NURBS Models

- Defined in parametric domain
- Directed closed polygon
- Loops
- Edges
- Segments
Multi-stage process

• Three stages
  – Proximity
  – Local Tracking
  – Contact
• Helps manage overall scene complexity
Global Minimum Distance Framework

- Use a hierarchical refinement method (LUB-tree)
  - completely stable
  - reasonable speed (20-100 Hz)
- Experimenting with local methods
  - 1000 Hz for point to trimmed model
  - Some stability concerns
LUB-Tree Framework

- LUB-Tree
  - Bounding hierarchy
  - Establish upper bound on minimum distance
  - Measure lower bound for current bounding volumes
  - Prune away impossible areas
  - Refine
LUB-Tree: Example

- Bounding volumes around model
LUB-Tree: Example

• Establish upper bound on minimum distance
LUB-Tree: Example

- Measure lower bound for current bounding volumes
LUB-Tree: Example

- Prune regions further away than the upper bound
LUB-Tree: Example

- Refine remaining regions
- Repeat
LUB-Tree: NURBS

• Apply polygonal style method to NURBS models
LUB-Tree: NURBS

- Refinement
  - Group remaining polynomial pieces
  - Extract
  - Refine
Time-critical Properties

• Upper and lower bounds found quickly
Local Tracking

• Initialized by global method
• Local methods
  – faster
  – give the desired behavior
Direct Parametric Tracking (DPT)

Initial State  Movement  Project  Compute parametric change and new surface position
DPT derivation for a curve

• B-spline curve

\[ \gamma'(u) = \frac{d\gamma}{du} \approx \frac{\Delta \gamma}{\Delta u}. \]

• Velocity curve relates movement along curve to change in parameter

\[ |\Delta u| \approx \frac{||\Delta \gamma||}{||\gamma'(u)||}. \]
DPT derivation (cont.)

• Use linear approximation

\[
\Delta \gamma \approx \frac{\langle \psi, \gamma'(u) \rangle}{\|\gamma'(u)\|^2} \gamma'(u).
\]

• Tangents are efficiently evaluated

\[
\gamma'(u^*) = \frac{(k - 1)}{u_{i^*} + k - u_{i^*} + 1} (P_{i^*+1} - P_{i^*}).
\]
DPT derivation (cont.)

• All together

\[
\Delta u \approx \frac{\langle \psi, (P_{i+1} - P_i) \rangle}{\|P_{i+1} - P_i\|^2} \left( \frac{u_{i+k} - u_{i+1}}{k - 1} \right).
\]

• 1000’s of Hz
• Pretty reliable
Higher-order tracking

• Orthogonal to tangents

\[
\mathbf{F} = (S(u, v) - P) \cdot \frac{\partial s}{\partial u} = 0 \\
(S(u, v) - P) \cdot \frac{\partial s}{\partial v} = 0
\]

• Multidimensional Newton’s method

\[
\mathbf{J} \cdot \Delta \mathbf{x} = -\mathbf{F}
\]

\[
\begin{bmatrix}
\frac{\partial}{\partial u}[(S-P) \cdot S_u] \\
\frac{\partial}{\partial v}[(S-P) \cdot S_u] \\
\frac{\partial}{\partial u}[(S-P) \cdot S_v] \\
\frac{\partial}{\partial v}[(S-P) \cdot S_v]
\end{bmatrix}
\begin{bmatrix}
\Delta u \\
\Delta v
\end{bmatrix} =
\begin{bmatrix}
(S-P) \cdot S_u \\
(S-P) \cdot S_v
\end{bmatrix}
\]
Haptic “Philosophy”

• Coherence can save you!
• Perform one iteration of Newton’s method at higher rates rather than more iterations at lower rates.
Improved stability

DPT method

Higher-order method
Dealing with Trimming Loops

• Brute force approach not feasible
• Grid approach
  – Boundary on bounding box
  – Walk algorithm
Grid Data

- **Goblet**
  - 3 surfaces
  - 254 segs
  - 13 max
  - 3 mean
  - 89% empty

- **Gear**
  - 22 surfaces
  - 1256 segs
  - 15 max
  - 4 mean
  - 92% empty

- **Crank Shaft**
  - 73 surfaces
  - 412 segs
  - 36 max
  - 4 mean
  - 89% empty
Current Projects

• Develop systems of equations for two models
  – minimum distance
  – penetration depth

Current Projects

- Apply concepts from NURBS local tracking back to polygonal models
- Encapsulate directional information in data structure
- Prune based on orientation, not spatial bounds

normal cone
view cone
Local minimum distance

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<th></th>
<th>Sphere</th>
<th>Holes3</th>
<th>Small bunny</th>
<th>Large bunny</th>
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Local minimum distance

- Also for model-model distance
Acknowledgements

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