Carrier Phase Measurements Characteristics and Utilization Overview

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• Observation Equation

$$ p = \rho + d\rho + c (dt - dT) + d_{\text{ion}} + d_{\text{trop}} + \varepsilon(p) $$

$$ \rho = \| \mathbf{r} - \mathbf{R} \| $$, where \( \mathbf{R} (x, y, z) \) is the (unknown) position vector of the observation point and \( \mathbf{r} \), that of the satellite observation point

- \( d\rho \) ... orbital error
- \( dt, dT \) ... satellite & receiver clock errors
- \( d_{\text{ion}}, d_{\text{trop}} \) ... ionospheric & tropospheric delays
- \( \varepsilon(p) \) ... \( f\{\varepsilon(p_{rx}), \varepsilon(p_{\text{mult}})\} \)
- \( \varepsilon(p_{rx}) \) ... receiver noise (Gaussian)

$$ \varepsilon(p_{rx[c/a]}) \approx 10 - 300 \text{ cm} $$
$$ \varepsilon(p_{rx[P]}) \approx 10 - 30 \text{ cm} $$
$$ \varepsilon(p_{\text{mult}}) \leq 1 \text{ chip (non-Gaussian)} $$
CARRIER PHASE OBSERVABLE

Observation equation (in metres)

\[ \Phi = \rho + d\rho + c(dt-dT) + \lambda N - d_{\text{ion}} + d_{\text{trop}} + \varepsilon_{\Phi} \]

where

\[ \rho \] geometric range (i.e. \( ||r-R|| \))

\[ d\rho \] orbital error

\[ dt \] satellite clock error

\[ dT \] receiver clock error

\[ N \] cycle ambiguity (integer number)

\[ d_{\text{ion}} \] ionospheric delay

\[ d_{\text{trop}} \] tropospheric delay

\[ \varepsilon_{\Phi} \] noise (\( \varepsilon_{\Phi_{\text{rx}}} + \varepsilon_{\Phi_{\text{multipath}}} \))

\[ \varepsilon_{\Phi_{\text{rx}}} \approx 1 - 5 \text{ mm} \]

\[ \varepsilon_{\Phi_{\text{multipath}}} \leq 0.25\lambda \]
GPS ERROR SOURCES

ERROR MAGNITUDE

• Satellite errors (1σ):
  Orbit & clock: < 5 m

• Propagation errors:
  Ionosphere: < 10 m
  Troposphere: 0.2 - 1.0 m

• C/A code receiver errors
  Code Multipath: < 5 m
  Code Noise: < 1 m
  Carrier Multipath: < 50 mm
  Carrier Noise: < 3 mm
SINGLE REFERENCE STATION DGPS CONCEPT

- Advantages
  (i) Reduction/elimination of following errors
  - Orbital (reduced)
  - Ionosphere and troposphere (reduced)
  - Satellite and receiver clock errors (eliminated)
  (ii) Better quality control
- Remaining errors
  - receiver noise (amplified)
  - multipath (amplified)
  - differential troposphere
  - differential ionosphere (for single frequency users)
WHY DIFFERENTIAL GPS?

• to decrease spatially correlated errors, namely
  - orbital errors
  - atmospheric errors

• to eliminate satellite ($\nabla$ differencing) and receiver clock errors
  ($\Delta$ differencing)

• site dependent errors (noise and multipath) are not reduced but amplified

• DGPS is required to fully exploit the high accuracy (cm-level) of carrier phase measurements, even in the absence of Selective Availability

• DGPS can also improve reliability, depending on system specifications
**BETWEEN RECEIVER SINGLE DIFFERENCE (Δ)**

- Concept: Subtract pseudorange at reference station ($R_1$) from that at remote ($R_2$):

  \[ \Delta = (\cdot)_{rx_2} - (\cdot)_{rx_1} \]

\[ \Delta p = \Delta \rho + \Delta \rho' - c \Delta T + \Delta_{ion} + \Delta_{trop} + \varepsilon_{\Delta p} \]

- Reduces orbital and atmospheric errors
- Eliminates satellite clock error, $dt$
- Does not reduce $\varepsilon_{\Delta p}$

**Method used for real-time applications:**
- Filtered $\Delta \rho$'s and $\Delta \rho/dt$'s are transmitted from $R_1$ to $R_2$ at regular intervals:
- RTCM SC104 Specifications: 50 - 100 bps data rate
- Positioning accuracy: 0.5 - 5 m
MOTIVATION

- Merge 'absolute' pseudorange capability and 'relative' carrier phase capability - pseudorange is not ambiguous but noisy, carrier phase is ambiguous but accurate
- Provides an alternative to pure pseudorange observations and is used in virtually all rx's firmware

METHODOLOGY

- Recursive filter to progressively increase weight on $\Phi$ while decreasing weight on $P$

- For example, smoothed pseudorange $\tilde{P}_k$ at time $k$:

\[
\tilde{P}_k = W_{P_k} P_k + W_{\Phi_k} \{ \tilde{P}_{k-1} + (\Phi_k - \Phi_{k-1}) \}
\]

- $W_{P_k}$ and $W_{\Phi_k}$ are the weights on the measured pseudorange and carrier phase components
CARRIER PHASE SMOOTHING (2/2)

\[ W_{P_k} = W_{P_{k-1}} - 0.01 \quad \{\text{e.g., } 0.01 \leq W_{P_k} \leq 1.00\} \]

\[ W_{\Phi_k} = W_{\Phi_{k-1}} + 0.01 \quad \{\text{e.g., } 0.00 \leq W_{\Phi_k} \leq 0.99\} \]

- At initialization (t_1)
  \[ \tilde{P}_{t_1} = P_1 \quad \{W_{P_{t_1}} = 1.0; W_{\Phi_{t_1}} = 1.0 - W_{P_{t_1}} = 0.0\} \]

- Use parallel filters to deal with code/cARRIER divergence and make the procedure more robust

SATELLITE/RECEIVER DOUBLE DIFFERENCE ($\Delta \nabla$)

\[ \Delta \nabla = \{( \bullet )_{\text{sat2}} - ( \bullet )_{\text{sat1}}\}_{\text{rx2}} - \{( \bullet )_{\text{sat2}} - ( \bullet )_{\text{sat1}}\}_{\text{rx1}} \]

\[ \Delta \nabla p = \Delta \nabla \rho + \Delta \nabla d\rho + \Delta \nabla d_{\text{ion}} + \Delta \nabla d_{\text{trop}} + \Delta \nabla \varepsilon(p) \]

\[ \Delta \nabla \Phi = \Delta \nabla \rho + \lambda \Delta \nabla N - \Delta \nabla d_{\text{ion}} + \Delta \nabla d_{\text{trop}} + \Delta \nabla \varepsilon(\Phi) \]

- Assumes that all observations are taken at same time
- Reduces orbital and atmospheric errors
- Eliminates satellite and receiver clock errors $dt$ and $dT$
- Does not reduce $\varepsilon(\rho)$'s or $\varepsilon(\Phi)$'s
- Ambiguity term $\Delta \nabla N$ can be held to an integer value for short baselines/monitor-remote separations:
  \[ \Delta \nabla \Phi = \Delta \nabla \rho + \lambda \Delta \nabla N + \Delta \nabla \varepsilon(\Phi) \]
OTF AMBIGUITY RESOLUTION (1/2)

• OTF: On-The-Fly, i.e., in kinematic mode
• Resolution of \((\Delta \nabla \rightarrow \text{double difference})\) integer ambiguities using a search technique while remote receiver is moving
• Real-time or post-mission batch algorithm
• Assumes that no irrecoverable cycle slips occur during the resolution period
• Numerous methods have been developed since early 80s, e.g., Ambiguity Function Method, least-squares search, FASF, FARA, lambda, etc.
  - methods explicitly/implicitly assume a known stochastic behavior for unmodelled errors (e.g., noise, multipath, differential atmospheric effects)
OTF AMBIGUITY RESOLUTION (#2/2)

• The time to resolution is a function of:
  - Use of $L_1$ vs $L_1 - L_2$ (widelane) observable
    ($\lambda_{L_1} = 19$ cm, $\lambda_{L_1-L_2} = 86$ cm)
  - distance between reference and remote receivers
  - number and geometry of satellites
  - differential atmospheric conditions
  - multipath conditions, code and carrier phase noise
  - ambiguity search method used
  - level of (statistical) reliability required