Optimizing Supply Chains in Military Operations

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Preliminaries

• Military operations are rare events;
• During peacetime the military consumes resources;
• Peacetime SC is similar to a business SC (MinCost, best business practices, efficiency);
• Strategic decisions regarding wartime SC are taken during peacetime (national supply levels, logistics force structure, doctrine), \( F(\text{threat, national capabilities}) \);
• Operational and tactical decisions are taken during wartime: \( F(\text{theater, scenario}) \).
Logistics Support Chain

Planning, implementing and controlling:

- **Supply**
- Mobilization
- Ordnance
- Maintenance
- Medical

**Strategic:** national resources and capabilities

**Operational:** theater-level deployment and employment

**Tactical:** Combat unit’s logistics

- Decision levels
- Sections of the SC
The General Problem

Design a supply chain that best responds to the battlefield needs during a military operation in a given theater (strategic and operational aspects)
Overview

• Characteristics of a military supply chain during an operation
• Comparing Mil. SC with Bus. SC
• Types of uncertainty in Mil. SC
• Descriptive model
• Prescriptive model
Logistics Supply Chain

- complete $k$-array rooted tree, $k=3-5$
- Main problem: how much to allocate (deploy) to each unit
- $F(\text{force, theaters of operations, enemy, threat, doctrine})$
Trendy Buzz-Terms and Tenets

- Velocity management
- Minimum footprint
- Just-in-case vs. just-in-time
- Trade mass for velocity

Minimize Tail, Maximize Response
## Retail vs. Military Supply Chains

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Retail</th>
<th>Military</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operation</td>
<td>routine, long-term, S - L</td>
<td>rare, short-term, XL-scale</td>
</tr>
<tr>
<td>Environment</td>
<td>neutral</td>
<td>hostile</td>
</tr>
<tr>
<td>Uncertainty</td>
<td>demand, cost, lead time</td>
<td>+ deployment, survival</td>
</tr>
<tr>
<td>Cost Consideration</td>
<td>mostly economical</td>
<td>mostly operational</td>
</tr>
<tr>
<td>Graph of LogNet</td>
<td>static</td>
<td>dynamic</td>
</tr>
<tr>
<td>Flow</td>
<td>sparse, (trucks)</td>
<td>massive, (convoys)</td>
</tr>
<tr>
<td>Modeling Approach</td>
<td>microscopic</td>
<td>macroscopic</td>
</tr>
<tr>
<td>Service Level Measures</td>
<td>relatively relaxed</td>
<td>relatively strict</td>
</tr>
</tbody>
</table>

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Service Level Measures

<table>
<thead>
<tr>
<th>Retail</th>
<th>Military</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pr[d_i \text{ is satisfied}] \geq 0.95, i,\ldots,n$</td>
<td>$\Pr[d_i \text{ is satisfied, } i=1,\ldots,n] \geq 0.95$</td>
</tr>
<tr>
<td>➞ “On average 95% of customers are satisfied all the time”</td>
<td>➞ “All customers are satisfied at least 95% of the time”</td>
</tr>
<tr>
<td>➞ Expected Value</td>
<td>➞ Chance Constraint</td>
</tr>
</tbody>
</table>
A Simple Risk Pooling Example

XX
XX
A
B
C

L, M, H

1K, 3K, 10K

Day

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>M</td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>L</td>
<td>L</td>
<td>L</td>
</tr>
</tbody>
</table>

100% response ➔ Corps base-stock = 3 × 3 × 10K = 90K

100% response ➔ Corps base-stock = (3 + 2) × 10K = 50K
Types of Uncertainty

- Delivery Uncertainty
- Initial Scenario Uncertainty
- Transition Uncertainty
- Demand (statistical) Uncertainty
Descriptive (Static) Approach

\[ d_{3j} = \text{Demand at node (3,j), } j=1,\ldots,6. \]
\[ d_{3j} \sim F_j \]
\[ x_{ij}(A) (x_{ij}(B)) = \text{Inventory deployed at node (i,j) in Plan A(B).} \]
\[ p_i = \text{Level } i \text{ timeliness probability} \]
Descriptive (Static) Approach cont.

\[ a_{2j} = \text{Max} \{0, d_{2j} - x_{2j}\}; \quad d_{2j} = \sum_{k \in S(2,j)} a_{3k} \]

\[ d_{11} = \sum_{k=1,2} a_{2k} \]

\[ a_{3j} = \text{Max} \{0, d_{3j} - x_{3j}\} \]

Sufficiency Probabilities:

\[ R_1 = R(x_{11}, x_{21}, \ldots, x_{36}) = \Pr[ d_{11} \leq x_{11} ] \]
\[ R_2 = R(x_{21}, \ldots, x_{36}) = \Pr[ d_{21} \leq x_{21}, d_{22} \leq x_{22} ] \]
\[ R_3 = R(x_{31}, \ldots, x_{36}) = \Pr[ d_{3j} \leq x_{3j}, j = 1, \ldots, 6 ] \]
Descriptive (Static) Approach cont.

Response Probability: \( Q = p_1R_1 + p_2(1-p_1)R_2 + p_3(1-p_1)(1-p_2)R_3 \)

(Plan A) \( P \) (Plan B) iff \( Q_A > Q_B \)

Example

A

\[
\begin{array}{c}
X \\
\downarrow P \\
0 \\
d_1 \\
0 \\
d_2 \\
\end{array}
\]

B

\[
\begin{array}{c}
0 \\
\downarrow P \\
X_1 \\
d_1 \\
X_2 \\
d_2 \\
\end{array}
\]

A \( P \) B iff

\[
p \geq \frac{\Pr[d_1 \leq X_1, d_2 \leq X_2]}{\Pr[d_1 + d_2 \leq X]}
\]

More details & examples:

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Prescriptive (Dynamic) Approach
(Inter-Temporal Network)

\[ d = [d(D1), d(E1), \ldots, d(F3), d(G3)] \]
Features of ITN

• Nodes and edges may be added and removed
• Slopes of diagonal edges represent lead-times
• Scenarios (demand vector, graph topology) are simulated
• Initial supply levels - $F(\text{initial demand, transportation needs, nodes/edges survivability})$
• Specific replenishing decision rules may be explicitly represented
Example: Two Periods Two Levels

\[
\begin{align*}
1 & \quad 2 \\
\begin{array}{c}
\bigcirc x \\
d_{11}, \ldots, d_{n1}
\end{array} & \quad \begin{array}{c}
\bigcirc x \\
d_{12}, \ldots, d_{n2}
\end{array} \\
\bigcirc y \\
y_1, \ldots, y_n & \quad p
\end{align*}
\]

\[
x = ? \\
y = ?
\]

\[
\text{Min } nc_1 x + c_2 y \\
\text{st} \\
\Pr[ d_{1j} \leq x, j = 1, \ldots, n ] \geq Q_1, \\
\Pr[ d_{2j} + d_{1j} \leq x + y_j, j = 1, \ldots, n ] p + \Pr[ d_{2j} + d_{1j} \leq x, j = 1, \ldots, n ](1 - p) \geq Q_2 \\
\sum_{j=1}^{n} y_j - y \leq 0 \\
x, y, y_j \geq 0
\]
CCP – Deterministic Equivalent

\[(d_{t1},...,d_{tn}) \sim F_t ; \quad G = F_1 \ast F_2\]

Min \( nc_1 x + c_2 y \)

\[st\]

\[x \geq F_1^{-1}(Q_1)\}

\[pG(x + y_1, ..., x + y_n) + (1 - p)G(x, ..., x) \geq Q_2\]

\[\sum_{j=1}^{n} y_j - y \leq 0\]

\[x, y, y_j \geq 0\]
Recourse

\[\begin{align*}
\min & \quad nc_1 x + c_2 y \\
\text{subject to} & \quad x \geq \{F_1^{-1}(Q_1)\} \\
& \quad \sum_{j=1}^{\infty} p \int_{0}^{F_2(x + y_j(d_{1j}) - d_{1j}/d_{1j})} dF_1(d_{11}, \ldots, d_{1n}) + (1-p)G(x) \geq Q_2 \\
& \quad \sum_{j=1}^{n} y_j(d_{1j}) - y \leq 0 \quad \text{for all possible vectors } (d_{11}, \ldots, d_{1n}) \\
& \quad x, y, y_j(d_{1j}) \geq 0
\end{align*}\]
Looking At Scenarios

• Scenario at Period $t$: $\{d_{t_1}^s, d_{t_2}^s, \ldots, d_{t_n}^s\}$, $t=1,2$
• $s_1 = 1, \ldots, S_1$; $s_2 = 1, \ldots, S_2$
• $\Pr[s_1] = q_1(s_1)$; $\Pr[s_2] = q_2(s_2)$

First Period

Min $x$

$x - u_{s_1} d_{1j}^s \geq 0 \quad j = 1, \ldots, n, \quad s_1 = 1, \ldots, S_1$

$\sum_{s_1=1}^{S_1} u_{s_1} q_1(s_1) \geq Q_1$

$x \geq 0, \quad u_{s_1} \in \{0,1\}$
The Two-Periods Problem

\[
\begin{align*}
\text{Min} & \quad n c_1 x + c_2 y \\
\text{st} & \\
\text{Period I} & \\
(1) & \quad x - u_{s_1} d_{1j} \geq 0 \quad j = 1, \ldots, n, \quad s_1 = 1, \ldots, S_1 \\
(2) & \quad \sum_{s_1=1}^{S_1} u_{s_1} q_1(s_1) \geq Q_1 \\
\text{Period II} & \\
(3) & \quad x - d_{1j} + y_{s_1} - d_{2j} + (1 - u_{s_1, s_2}^+) M \geq 0 \\
(4) & \quad x - d_{1j} - d_{2j} + (1 - u_{s_1, s_2}^-) M \geq 0 \\
(5) & \quad p \sum_{s_2=1}^{S_2} q_{2j}(s_1, s_2) u_{s_1, s_2}^+ + (1 - p) \sum_{s_2=1}^{S_2} q_{2j}(s_1, s_2) u_{s_1, s_2}^- \geq Q_2, \quad s_1 = 1, \ldots, S_1 \\
(6) & \quad \sum_{j=1}^{n} y_{s_1} - y_{s_i} \leq 0 \quad s_1 = 1, \ldots, S_1 \\
(7) & \quad y_{s_1} - y \leq 0 \\
x, y, y_{s_1}, y_{j} \geq 0, \quad u_{s_1, s_2}, u_{s_1, s_2}^+, u_{s_1, s_2}^- \in \{0,1\}
\end{align*}
\]
Summary

• Military supply chains are rare;
• Supply-related risks are very high and must be explicitly modeled;
• The Logistics network has a dynamic structure;
• Several facets of uncertainty;
• Descriptive and prescriptive models;
• Strict probabilistic criteria;
• Recourse modeling is important and useful (SP).