Managing High-Tech Capacity via Reservation Contracts

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*Joint work with Murat Erkoc*
An U.S. Telecommunications Component-Manufacturer

- Client Systems
  - Enterprise/Metro Wireless Access
- Metro/Regional Transport
- Long-Haul Backbone
Rapid innovation & volatile demand

High obsolescence rate on technology

Short product life-cycle (avg. 18 months)

Capital intensive facilities ($2 Billion+)

High capacity costs

Manufacturers are conservative in capacity expansion of any form

Capacity reservation contracts
Some Background

- Capacity Planning vs. Capacity Reservation
- Optimization vs. Characterization
- Solution vs. Process

The main motivations?
- how to price capacity reservation?
- to go over the thought process analytically
- preparing for the “soft capacity” environment
High-Tech Supply Chain Building Blocks

Contract Manufacturers

CM 1
CM 2
CM n

Supplier 1
Supplier 2
Supplier K

Buyer 1
Buyer 2
Buyer M

Spot Market

Capacity Reservation Contract
Capacity Reservation: *Settings and Assumptions* (Erkoc and Wu, 2002a)

- Between the supplier and a major customer
- Stochastic market demand
- Capacity must be built in advance (tactical)
  - outsourcing, work force expansion, procurement of raw material, or pre-processing of semi-finished products
- Convex capacity cost (considers residual value)
- No inventory carry-over (single period)
- Complete information (cost, demand distribution)
Product Life-Cycle

- Design Win
- Final Design
- Production Ramp
- Supply Commitment Decision
- Prototypes
- Models
- Uncertain Supply Requirements

Time
Related Literature

- **Buyer’s perspective:** Silver and Jan (1994), Jan and Silver (1995), Brown and Lee (1998)
- **Supplier-Buyer perspectives:** Cachon and Larivier (2001), Tomlin (1999), Serel et. al. (2001)
- **Options:** Donohue (2000), Barnes-Schuster et. al. (2000)
- **Main Distinctions**
  - to reserve or not to reserve
  - wholesale price is not a contract parameter
  - linear vs. convex capacity cost
  - forced vs. voluntary compliance
  - buyer vs. supplier led channel
Integrated Channel

\[ S_F(k) = k - \int_{0}^{k} F(x) \, dx \]

expected sales given capacity \( k \)
Delegation of Control in the Supply Chain

- **System’s Profit (concave)**
  \[ \Pi_I = (p - c) \cdot S_F(k) - V(k) \]

- Let \( k^o \) be the system optimal solution
  \[ k^o = F^{-1} \left( \frac{v^o}{p - c} \right) \]

- **Supplier’s newsvendor profit function**
  \[ \Pi_s^0 = (w - c) \cdot S_F(k) - V(k) \]

- Let \( k^* \) be the newsvendor solution
  \[ k^* = F^{-1} \left( \frac{v^*}{w - c} \right) \]

- \( k^* < k^o \)
Due to *double marginalization*, the system optimal capacity is always greater than the supplier’s newsvendor capacity.
Reservation Contract with Deductible Fee

- For each unit of capacity reserved, the buyer is charged a fee.
- The fee is later deducted from the purchasing price if the reserved capacity is used by the intended customer.
- If the buyer’s demand exceeds the reservation amount, the orders are satisfied based on the availability of “extra capacity.”
- Customer reservations are locked in and some extra capacity may be built during expansion.
- The wholesale price $w$ and product cost $c$ are set at the “design-win” phase, which are not negotiable.
The Sequence of Events

Supplier announces reservation fee ($r$)

Supplier decides how much capacity to build ($k$)

Buyer announces how much to reserve ($q$)

Demand is realized ($x$)

Buyer is penalized for unused reserved capacity
Supplier’s Capacity Decision

\[
\text{Max} \ (w - c)S_F(k) - V(k) + r \cdot E(\max(0, q - x))
\]

s.t.

\[k \geq q\]

If \[q \geq k^*\], then the constraint must be binding
Buyer’s Profit

Buyer profit function is conditional due to the fact that he never reserves less than $k^*$

\[ \Pi^0_B = \begin{cases} 
(p-w)S(q) - r \int_0^q F(x) \, dx & \text{if } q > k^* \\
(p-w)S(k^*) & o / w 
\end{cases} \]

- In optimality
  \[ F(q^*) = \frac{p-w}{p-w+r} \]
- Reservation quantity $q$ decreases in reservation fee
Buyer’s Incentive to Reserve

- Buyer never reserves less than $k^*$. In fact, she reserves only if the reservation fee is below a certain threshold, $r^*$. The reservation quantity will be at least $q^*$, where

$$\Pi_B(q^*) = \Pi_B^0(k^*)$$

- $q^*$ is a function of $w$ and does not depend on $p$
- The buyer reserves iff $r \leq r^*$

Threshold fee

$$r^* = (p - w) \frac{\bar{F}(q^*)}{F(q^*)}$$

Reservation Qty.
Supplier’s Profit

The reservation quantity determines the final capacity, so we may rewrite the profit function as

$$\max \Pi_s = (w - c)S(q) - V(q) + (p - w) \frac{\overline{F}(q)}{F(q)} \int_0^q F(x)dx$$

$$q^{\text{max}} = \max(q^t, q^w)$$

Supplier’s profit function is unimodal if the demand distribution has increasing failure rate (IFR)

The function is strictly decreasing in $[k^o, \infty)$
The Supplier’s Faces Three Profit Scenarios in sequence as the Buyer’s Margin Decreases

Theorem: It is individually rational for the supplier and the buyer to enter the reservation contract if the buyer revenue margin \( p \) is no less than the threshold \( p^t \) as follows:

\[
p^t = c + \frac{V(q^t) - V(k^*)}{S(q^t) - S(k^*)}
\]

\[
\Rightarrow \Pi_I(q^t) > \Pi_I(k^*)
\]

The buyer’s margin must be sufficiently high to justify capacity reservation.
Impact of Market Size and Variability: Shifted Family Distributions

If the distribution is from a “shifted” family

\[ F(x \mid \Theta) = F(x - \Theta \mid 0) \]

\[ r^* = \min(r^t, w) \]

the standard deviation remains the same as the market size increases

With linear capacity cost

- the threshold reservation fee \( r^t \) does not change in market size \( \Theta \)
- the threshold reservation quantity \( q^t \) does increase in \( \Theta \) (literature)

With convex capacity cost,

- both the threshold fee \( r^t \) and quantity \( q^t \) increase in market size \( \Theta \)
- there exists a threshold \( \Theta^t \) over which reservation is not “win-win”
- the decrease in the coefficient of variation due to market size may not sufficient to offset the risk of expansion.
Impact of Market Size and Variability: Scaled Family Distributions

\[ F(x | \theta) = F\left(\frac{x}{\theta} | 1\right) \]

- the standard deviation increases with the market size, thus, the threshold reservation fee \( r' \) increases in market size \( \theta \).
- even with linear capacity cost, the increase in reservation payment (due to market size) is not sufficient to balance the increase in risk.
Capacity reservation may create surplus for the channel.

However, except for a few special cases the surplus is sub-optimal.

We design two additional coordination mechanisms to achieve optimality:

- Partial-deduction (PD) contract
- Cost-sharing (Options) contract
Suppose that the supplier offers a contract \((r, r_2)\) where the reservation fee is \(r\) and the deductible portion is \(r_2\)

\[
r_2 = \frac{p - c}{v^o} \left[ r - (p - w) \right], \text{ but } r > r_2 > 0, \text{ thus}
\]

\[
(p - w) \frac{v^o}{p - c - v^o} \geq r \geq (p - w) \frac{v^o}{p - c}
\]

**Theorem.** Under PD contract the buyer will reserve the channel optimal capacity, i.e., \(q(r) = k^0\). Moreover,

1. The buyer's profit is increasing in \(r\),
2. The supplier's profit is decreasing in \(r\),
3. The supplier would only offer such contract if \(v^0 \leq 2w - c\)
Cost-Sharing Contracts: *Capacity Options*

Suppose the supplier proposes a contract where the buyer pays $\alpha V(Q)$ for reserving $Q$ units (options) whereas the supplier returns $r_2$ for each exercised reservation quantity. The channel is coordinated if

$$r_2 = \alpha (p - c) - (p - w) \text{ and } \frac{\Pi^0_B}{\Pi^*_I} \leq \alpha \leq \frac{\Pi^*_I - \Pi^0_S}{\Pi^*_I}$$

Supplier gets $(1 - \alpha) \Pi^*_I$  \quad Buyer gets $\alpha \Pi^*_I$

Contract setting is independent of the demand distribution
Capacity Reservation with Buyer Competition

(Erkoc and Wu, 2002b)

- Introducing horizontal buyer competition
- Still a supplier-lead single-period model
- Two identical buyers from independent markets competing for the capacity
- The demand distribution for buyer $i$ is $G_i(x)$
- The supplier offers a reservation fee, $r$, for each unit of capacity that is uniform across buyers
- Excess capacity, if any exists, is distributed proportional to the buyers’ reservation quantities
The buyers’ competition will be

- a Fixed Capacity Reservation (FCR) Game, if (total reservation amount) \( \leq k^* \)
- a Variable Capacity Reservation (VCR) Game, otherwise
Fixed Capacity Reservation Game (FCR)

- Capacity is fixed at $k^*$
- A zero-sum game
- At equilibrium the buyers are compelled to reserve
- Capacity allocated to buyer $i$:

\[ h_i = \frac{q_i}{q_i + q_j} k^* \]

- With identical buyers

\[ r = \frac{(p-w)}{4q^k G(q^k)} k^* \frac{1}{2} \]
### Capacity Allocation

<table>
<thead>
<tr>
<th>Condition</th>
<th>Constraint</th>
<th>Description</th>
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<tbody>
<tr>
<td>$x_i \leq h_i$</td>
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**Diagram:**
- $q_{-i}$
- $q_i$
- $x_i \leq h_i$
- $x_i > h_i, x_{-i} \leq k^* - h_i$
- $x_i > h_i, x_{-i} > k^* - h_i$
- $x_i > h_i, x_{-i} > h_{-i}$

**Legend:**
- Buyer $i$ demand, $x_i$
- Buyer $-i$ demand, $x_{-i}$
Variable Capacity Reservation (VCR) Game

- Capacity is determined by total reservations:
  \[ k = q_i + q_{-i} \]

- Buyers’ profit function:
  \[ \Pi_B = (p - w)S_q(q_i, q_{-i}) - r \int_0^{q_i} G(x)dx \]

- Optimal buyer profits decrease in reservation fee

- At any point, each buyer wishes the other to reserve more
Game within a Game: FCR or VCR?

In equilibrium, the buyers would prefer the VCR game so long as the reservation fee is below a certain threshold $r^e$. 

Equilibrium Reservations: $f(5, w=10, V(k)=5k, c=3, D\sim N(1, 1/4)$
If the total reservation amount is less than $k^*$, the supplier’s optimal capacity will be $k^*$ and

$$\Pi_s^{FCR} = (w - c)S(k^*) - V(k^*) + 2r_k(q^e) \int_0^{q^e} G(x)dx$$

Otherwise, the total reservation amount determines the final capacity and

$$\Pi_s^{VCR} = (w - c)S(2q^e) - V(2q^e) + 2r^{VCR}(q^e) \int_0^{q^e} G(x)dx$$
Supplier’s First Stage Model

\[
\begin{align*}
\text{Max} & \quad y \Pi_s^{FCR} + (1 - y) \Pi_s^{VCR} \\
\text{s.t.} & \quad q - (1 - y)q^t \geq 0 \\
& \quad q \geq q(w) \\
& \quad y \in \{0, 1\}
\end{align*}
\]

\( y=0 \) iff the surplus is created at the threshold reservation fee, i.e.,

\[\Pi_I(k^*) < \Pi_I(q^t)\]
The $N$-Buyer Case

- **Proportional allocation of excess capacity**
  - Becomes intractable
  - Existence of equilibrium can be established for a subset of *uplifting contracts*

- **Uniform allocation of excess capacity**
  \[
  S_{i}^{VCR}(q_i, q_{-i}) = S_{G}(q_i + Z_{-i}(q_{-i}, x_{-i}))
  \]
  - Equilibrium can be established via supermodularity
  - The equilibrium is unique
Conclusions

- Impact of capacity reservations on supply chain efficiency (when are they beneficial?)
- Convex capacity cost
- Effects of market size and demand variability
- Horizontal buyer competition
- Proposed Contracts for channel coordination
- Other Considerations
  - Voluntary Compliance
  - Partial demand information
- Working papers (Erkoc and Wu, 2002 a,b)
Future Considerations

- Multiple periods with inventory considerations
- Correlated demands and/or intra-market competition
- Sequential or repeated negotiations
- Multiple-supplier model
- In house production v.s. outsourcing
- Capacity Reservation from the *Contract Manufacturers* Perspective
Buyer’s Profit and Equilibrium

\[ S_k(q_i, q_{-i}) = \int_0^{h_i} xg(x)dx + \int_{h_i}^{K^*} xg(x)G(K^* - x)dx + \int_{h_i}^{\infty} \int_0^{h_{-i}} (K^* - y)g(y)g(x)dydx \]
\[ - \int_{h_i}^{K^*} \int_0^{K^* - x} (K^* - y)g(y)g(x)dydx + h_i \tilde{G}(h_i)G(h_{-i}) \]

At equilibrium both buyers reserve the same quantity, \( q^k \) yielding

\[ \Pi_B^e = (p - w)S_k(K^*/2, K^*/2) - r \int_0^{q^k} G(x)dx < \Pi_B^0 \]

\[ q^k G(q^k) = \frac{(p - w)}{4r} K^* \bar{G}^2 (K^*/2) \]
The Uplifting Contract (Spot Market Pricing)

- The supplier charges an extra $\rho$ for any purchase order beyond the reservation amount (spot market price)

$$\rho = \begin{cases} b + d & \text{if } q < k^*/2 \\ b & \text{o/w} \end{cases}$$

- The supplier offers a contract $(\rho, r)$

- The contract coordinates the channel if $b, d$ satisfy the following:

$$b(r) = \frac{rG(k^o/2) - \frac{1}{2}(p-w)\left(\bar{G}(k^o) + \bar{F}(k^o)\right)}{k^o/2}$$

$$d(r) = r \frac{G(k^*/2)}{G(k^*/2)} - b(r)$$

- With $d$ the supplier dictates if the buyers choose to play the VCR game

- By adjusting the reservation fee, the supplier may dictate which game (FCR or VCR) the buyers play