Exploration of Interest Rate Data

Philip H. Dybvig
Washington University in Saint Louis

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http://phildybvig.com
How should we model interest rates for pricing and risk mgmt?

- A specific parametric model is useful
  - sharp predictions
  - accurate estimation of parameters
- But, what parametric model to use?
- Approach taken today
  - nonparametric: little prior restriction on the model
  - exploratory: statistical analysis without formal statistics
  - uses a long daily series of 3mo Treasury discounts from the Fed
- Extends prior work I did around 1990
  - Interesting time for an extension given current Fed policy etc.
  - Surprisingly, none of the results change.
- Application: long bonds
  - work in FAJ with Bill Marshall
  - parameter uncertainty? matters
Figure 1

3mo T–Bill Discounts
daily data, 540104—040312

date

vol is higher when rates are higher
vol of log of rate is higher when rates are lower
Square Root of 3mo T–Bill Discounts
daily data, 540104---040312

vol of sqrt of rate is balanced but not constant
Figure 4 Except as noted, sample variance means squared change

Cumulative Sample Variance of 3mo T–Bill Discounts
daily data, 540104——040312

vol is higher when rates are higher
Figure 5

Sample Cum Var of Square Root of 3mo T–Bill Discounts
daily data, 540104—040312

vol of log of rate is higher when rates are lower
Sample Cum Var of Log of 3mo T–Bill Discounts
daily data, 540104—040312

vol of sqrt of rate is balanced but not constant
interest rate regimes?
Figure 7

Sample Cum Var of Square Root of 3mo T–Bill Discounts
daily data, 540104—040312

sanity check: cum sq resid looks the same as cum sq changes
Sample Var of Square Root of 3mo T–Bill Discounts
daily data, 540104—040312

sample var not piecewise constant: regimes?
Sample Var of Square Root of 3mo T–Bill Discounts
daily data, 540104—040312

not just sampling error: same pattern for 40-day samples
Log of Sample Var of Sqrt of 3mo T–Bill Discounts
daily data, 540104–040312

log sample var’s vol seems pretty constant
Figure 11

Cum Var of Log of Var of Sqrt of 3mo T–Bill Discounts
daily data, 540104—040312

cum var

0 200 500

date (20 days/interval)

confirms vol of vol of sqrt(discount) pretty nearly constant
Implications for the interest rate process (discount vs 3mo yield vs spot rate, bid vs ask etc. will not affect these conclusions)...

(1) \( r = x^2 \)
(2) \( dx = \mu_x dt + v dZ_1 \)
(3) \( dv = \mu_v dt + \sigma_v v dZ_2 \)

where \( \sigma_v \) is approximately constant and the means are mean-reverting, for \( v \) more obviously than for \( x \) (or \( r \)). If \( r = x^\alpha \),

(4) \( dr = \alpha (1 - \alpha) x^{\alpha - 2} v^2 dt + \alpha x^{\alpha - 1} dv \)
    \[ = \mu_r dt + \alpha vr^{(\alpha - 1)/\alpha} dZ_1 \]

or for \( \alpha = 2 \) as in (1),

(5) \( dr = \mu_r dt + 2v \sqrt{r} dZ_1 \).

where, if \( v \) is lognormal, so is \( 2v \).
Discount Bond Yields and Average Spot Rates
basic model with known parameters

pricing with a known parameter (long-term mean rate = 9%)...

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Figure 4 from Long Bonds paper in FAJ with Bill Marshall

Discount Bond Yields and Average Spot Rates model with unknown long-term average spot rate

...can be much different from pricing with a random parameter (long-term mean rate = 10% with probability .9 and 1% with probability .1)
Summary

- "Square root" process with lognormal stochastic vol seem to fit well
- Drift mean reverting
- Sample “square bracket” process is a useful diagnostic
- Parameter uncertainty can have a big impact on pricing longer bonds