Dogbert Explains Stock Investing
Pay Attention.

First, employees provide valuable data.

Is your project on schedule?
I didn’t know I had a project.

A manager refines the data.

We’re on schedule.

No problems whatsoever.

The CEO gives “visibility” to analysts.

Accountants publish bad news in footnotes using a combination of nanotechnology and gibberish.

Discount brokerage firms tell you that you’re smart.

Use your own ideas!

Investors do their own research.

Buy it because I did.

Thank you, unbiased stranger!

A secret society of Donald Trump look-alikes end up with all your money.

You’re fired.

You are!
Modeling liquidity, risk and transaction costs in the LSE using low intelligence agents

J. Doyne Farmer
Santa Fe Institute
Institute for Mathematics and its Applications
May 26, 2004

Research supported by:
McKinsey & Company  Bill Miller
Credit Suisse First Boston  Bob Maxfield
McDonnell Foundation  Prediction Company
Advertisement

• “Beyond Equilibrium and Efficiency”, J.D.F. and John Geanakoplos
  – Preliminary manuscript: reviews and critiques equilibrium theory in financial economics
  – Discusses alternatives
  – Warning: 180 pages

• If you are willing to read and possibly comment, send email to jdf@santafe.edu.
Goals

• Understand
  – risk (price volatility)
  – liquidity (impact of trading on prices)
  – transaction costs (spread)

• Find simple laws relating these to measurable properties of markets
Why economics is hard

• Must model strategic interaction of agents
• Standard solution
  – Selfish, rational utility maximizing agents at equilibrium
• Problems with standard solution
  – Lack of rationality
  – Lack of equilibrium
  – Intractability in non-trivial settings
  – Parameters cannot be independently measured
• No easy fix
  – How to avoid “wilderness of bounded rationality”?
Alternative

• Use extremely simple agent model (random behavior) to probe market institutions.
• Use this to understand what is dictated by market institutions alone.
  – Benchmark for agent intelligence
• Gradually introduce simple forms of bounded rationality, using (imperfect) arbitrage as a guide.
A few seminal papers illustrating zero intelligence

- Gary Becker, Irrational behavior and economic theory, J. of Political Economy 1, February (1962).
Three models

1. Analytically tractable zero intelligence model
2. Empirical model based on study of data
3. “First principles” model, exploiting arbitrage

Common features:
- Each has as model of order flow
- Price formation rules given by double auction
- Prices emerge from interaction of price formation rules and order flow model

- In some sense intermediate between “first principles” and purely econometric
Continuous double auction

Price adjustment in orders both to buy and to sell

Market operates continuously

Execution priority:

- Lower priced sell orders or higher priced buy orders have priority
- First order placed has priority when multiple orders have same price.
Patient trading

- Patient traders place **non-marketable limit orders** that do not lead to an immediate transaction
- Non-marketable limit orders accumulate
- Limit order book is a storage device
Impatient trading

**Market order:**
- An order to buy or sell up to a given volume
- No limit price is defined
- Executed immediately
- Often causes unfavorable price impact

<table>
<thead>
<tr>
<th>Market Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>BUY / SELL</td>
</tr>
<tr>
<td># OF SHARES</td>
</tr>
</tbody>
</table>
Order cancellation

Limit order cancellations:
• Limit orders can be cancelled by the owner
• Market defined expiration
ZI model
(Unrealistic but somewhat tractable)

- **Limit order arrival**: Poisson process in time & price; $\alpha$
- **Market order arrival**: Poisson process in time; $\mu$
- **Cancellation**: random in time (like radioactive decay); $\delta$
- **Separate processes for buying and selling**, with same parameters.

**Depth profile** $n(p,t)$: Number of shares in limit order book at price $p$, time $t$. 

$$n(p,t)$$
Achievements of ZI model so far

• Dimensional analysis
  – (price, shares, time)
  – Rough predictions, simplified analysis
• Mean field theory and simulation
  – scaling laws relating order flow and prices
• A few good results in tests with real data
Comparison to LSE data

Ilija Zovko

Paolo Patelli
### London Stock Exchange data set

**Analysis based on automated (on book) orders only**

<table>
<thead>
<tr>
<th>Company name</th>
<th>Trade type indicator</th>
<th>Company code</th>
<th>Volume weighted average price of today's trading</th>
<th>Total of today's shares traded (order book only)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ABC Holdings</strong></td>
<td><strong>ABC</strong></td>
<td><strong>P Close</strong></td>
<td><strong>517 1/2</strong></td>
<td><strong>GBX</strong></td>
</tr>
<tr>
<td><strong>NMS</strong></td>
<td>200,000</td>
<td>Segment SET1</td>
<td>Sector FT10</td>
<td>ISIN GB000263494</td>
</tr>
<tr>
<td><strong>Last</strong></td>
<td><strong>524 1/2 AT</strong></td>
<td><strong>at 11.08</strong></td>
<td><strong>Vol 3,952</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Prev</strong></td>
<td>524</td>
<td>525 AT</td>
<td>524 1/4 AT</td>
<td>524</td>
</tr>
<tr>
<td><strong>Trade Hi</strong></td>
<td>530</td>
<td>Open</td>
<td><strong>520</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Trade Lo</strong></td>
<td>517</td>
<td>VWAP</td>
<td><strong>527</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Total shares traded</strong></td>
<td><strong>4.61m</strong></td>
<td>SETS Vol</td>
<td>2.58m</td>
<td></td>
</tr>
<tr>
<td><strong>Number of buy orders at the best price</strong></td>
<td><strong>BUY</strong></td>
<td>TVol 543,906</td>
<td>Base 520</td>
<td></td>
</tr>
<tr>
<td><strong>Yellow strip</strong></td>
<td></td>
<td>MOVol</td>
<td><strong>524 - 525</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Total volume of buy orders currently on the bid</strong></td>
<td><strong>BUY</strong></td>
<td>20,000</td>
<td>524</td>
<td>525</td>
</tr>
<tr>
<td><strong>Buy market order volume</strong></td>
<td></td>
<td>20,000</td>
<td>525</td>
<td>10,000</td>
</tr>
<tr>
<td><strong>Volume at best bid price</strong></td>
<td></td>
<td>50,000</td>
<td>526</td>
<td>101,900</td>
</tr>
<tr>
<td><strong>Cumulative market price &amp; volume information</strong></td>
<td></td>
<td>20,000</td>
<td>526</td>
<td>101,900</td>
</tr>
<tr>
<td><strong>Order price per share</strong></td>
<td><strong>BUY</strong></td>
<td>Base price</td>
<td><strong>524</strong></td>
<td><strong>525</strong></td>
</tr>
<tr>
<td><strong>Order price per share</strong></td>
<td></td>
<td>Best bid offer (the spread)</td>
<td>21,900</td>
<td>31,900</td>
</tr>
<tr>
<td><strong>Total volume of sell orders currently on the offer</strong></td>
<td><strong>SELL</strong></td>
<td>Tvol 702,746</td>
<td>MOVol</td>
<td></td>
</tr>
<tr>
<td><strong>Number of sell orders at the best price</strong></td>
<td></td>
<td><strong>525.00</strong></td>
<td>10,000</td>
<td>525.00</td>
</tr>
<tr>
<td><strong>Total volume of sell orders currently on the bid</strong></td>
<td></td>
<td>525.34</td>
<td>10,000</td>
<td>525.34</td>
</tr>
<tr>
<td><strong>Volume at best offer price</strong></td>
<td></td>
<td>20,000</td>
<td>525.74</td>
<td>525.74</td>
</tr>
</tbody>
</table>

**Currency**
- GBX = pence
- GBP = pounds
- EUR = euros

**International security number**

**Total of shares traded yesterday**

**Last order book trade price or indicative uncrossing price**

**Trade high and low share price (order book only)**

**Previous day’s closing price**

**Highest & lowest prices of the day on and off the order book**

**Last traded price**

**Last five trade prices**

**Normal market size**
Upstairs, downstairs

- Off-book (upstairs) trades account for 35% of transactions, about 50% of volume.
- Off-book trades sizes are power law distributed, on-book (downstairs) trade sizes are not.
- On-book trade sizes long-memory; off-book not*
- NYSE data (with on/off book mixed together) show same independence of order volume.
- Problems with mixing on and off book trades due to reporting time lag for off book trades.
- Which is more important for price formation?
Parameters of model

\[ \alpha = \text{limit order rate} \]
\[ \mu = \text{market order rate} \]
\[ \delta = \text{order cancellation rate} \]
\[ \sigma = \text{typical order size} \]
\[ dp = \text{tick size} \]

Order flow rates

Discreteness parameters

Three fundamental dimensional quantities:

*shares, price, time*
Testing prediction of spread

- From mean field theory:

\[ \frac{\mu}{\alpha} f\left(\frac{\sigma \delta}{\mu}\right) \]
Predicted price diffusion rate

\[ \mu^{5/2} \delta^{1/2} \sigma^{-1/2} \]

\[ \alpha^2 \]

http://www.dw-world.de/russian/0,3367,2212_A_985770_1_A,00.html

?????? ??????????? ????? ?????????? ?????? ??????????????? , ???????
?????????? ??????? ??????, ? ????????????????? ????? ???????????????
?????.?? ? ?????????? ????? ??????????????? ????? ??????? ??
?????????????? ? ??????? ??? ????? ??????????? ????? ???????????
??????????????, ?????????? ?????????????????? ????? ??????
?????????? ?? ????????????? ????????????? . ?????? ??????????????????? ???????
???????????? ?????? ????????? ????????? ????? ??????????? ?? ???????????.

?????????? ?????? ??? ??????????? ?????? ??????? ???????, 
???????????? ?????? ??????? (J. Doyne Farmer), ????????????? ?????????
????-?? ?? ??? ??? -? ????? . ? ? ????????????? ?? ??????????????? , ???
?????? ? ?????? ? ????? ??????????? ?? ????????????? ????????? ? ?????? ??
???????? ??????? ?? ?? ????? ??????????? ?????????????????, ???????????????
?????? ?????????? , ????????????????????????????????????? ??????? ??
?????? ? 1998-?? ?? 2000-? ???, ????????????????? ????????????? ???????

???????????
Volatility autocorrelation

Time correlation in financial time series
Fat tails in prices
Price fluctuations have fat tails

• Mandelbrot (63), Fama (64), …
  – Power law tails?
    \[ \lim_{x \to \infty} P(r > x) \approx x^{-\beta} \]

• Many theories:
  – Excessively rapid learning (SFI Stock market)
  – Heterogeneous beliefs (Lux & Marchesi, …)
  – Herding behavior (Cont and Bouchaud)
  – Minority game dynamics (Challet & Zhang, …)
  – Large market orders (Gabaix, Gopikrishnan, Plerou, Stanley)
  – …
Investigation of fat tails

Laszlo Gillemot

Szabolcs Mike

Fabrizio Lillo

Anindya Sen
Price changes are almost independent of volume

\[ P(r > x | V) \]

\[ r = \log p_{after} - \log p_{before} \]
A typical large price change
Gap distribution vs. price distribution
Tail exponent of rtns. Vs. gap

Return vs First Gap for 16 Stocks

Tail exponents of first gap distributions

Tail exponents of return distributions

slope = 0.979
Building a better empirical model for real order flow

• Real order flow is not random
  – Unconditional order placement distribution
  – Correlations in order flow
  – Dependence on spread
  – …
Autocorrelation of orders

- Prices are roughly uncorrelated
- Order sign, order volume, liquidity all long-memory
- Anti-correlations between signs vs. volume or liquidity
Unconditional order placement distribution

Ilija Zovko
Distribution of relative limit price conditioned on spread

Cumulative probability distribution, relative to opposite best price
Limit order placement

Probability density, relative to opposite best price
Empirical model

- Whether limit or market order depends on spread.
- Relative limit price depends on spread.
- Lifetime depends on where order is placed.
- Market orders remove all volume at best with 50% probability.
- Long-memory order flow.

Return distribution: red = real, black = model.
Ecology of arbitrage

Jim Girard

Jim Rutt

"I'll cannot smoke cigars in heaven, I shall not go." - Mark Twain
Market ecology

- As in biology, financial strategies have
  - Variation (innovation)
  - Selection
  - Propagation
- Agents form a specialized and diverse ecology.
- How to use this quantitatively?
Building a market ecology

• Exploit arbitrages of ZI model
• Liquidity demand is “food” for arbitrageurs.
• Agents in ecology of arbitrage.
  – Market maker
  – Order imbalance trader
  – Technical trader (not implemented yet)
Two low intelligence agents

• Market maker
  – Makes simultaneous bid and offer
  – Never goes below a minimum spread
  – Inventory limit

• Order imbalance trader
  – Places orders based on imbalance in order book
  – Threshold depends on (expected return - spread)
Profits vs. agent trading volume
The road to efficiency is not straight

- Efficiency comes about indirectly, and only through interaction of diverse specialized agents
- Order imbalance trader:
  - Exploiting order imbalance only makes it worse
  - Strategy is limited by widening spread
- Market maker:
  - Feeds off of order imbalance trader
  - Improves order imbalance
- MM + OI do not remove price anti-correlations
  - Need technical trader to make market efficient!
Practical implications

• Trading tactics
• Automated market making
• Market design:
  – To reduce volatility and spread: encourage limit orders, discourage market orders
  – Understanding determinants of large risks
  – Order granularity vs. tick size
  – Detecting poor markets
Conclusions

• ZI/LI models give benchmark for agent intelligence.
• Divide and conquer strategy
  – First understand how order flow affects market
  – Then understand what determines order flow
• Parameters are directly measurable*.
• To tractably model market institutions, it may be necessary to drop heavy baggage of equilibrium
  – ZI captures feedback between price formation and order flow
  – Empirical model does this more realistically
• Noise traders are economic users of market, supporting ecology of arbitrage
Upstairs, downstairs

- Off-book (upstairs) trades account for 35% of transactions, about 50% of volume.
- Off-book trades sizes are power law distributed, on-book (downstairs) trade sizes are not.
- On-book trade sizes long-memory; off-book not*
- NYSE data (with on/off book mixed together) show same independence of order volume.
- Problems with mixing on and off book trades due to reporting time lag for off book trades.
- Which is more important for price formation?
Price changes are almost independent of volume (NYSE)

\[ r = \log p_{\text{after}} - \log p_{\text{before}} \]

\[ P(r > x | V) \]
Market (price) impact

- Market impact is change in price caused by market order arrival. Instantaneous market impact is:
  - Price immediately after arrival - price immediately before
  - Closely related to supply and demand
Market impact function ("standard" units)
\[ \Delta \hat{p} = \frac{\Delta p \alpha}{\mu} \]

\[ \hat{N} = \frac{N \delta}{\mu} \]